Form, Use, and Sustainability:
A Geometric and Structural Feasibility Study of Hypar Shells

Nathan Brown

April 16, 2012

Submitted in partial fulfillment
Of the requirements for the degree of
Bachelor of Science in Engineering
Department of Civil and Environmental Engineering
Architecture and Engineering Program
Princeton University

I hereby declare that I am the sole author of this thesis.

I authorize Princeton University to lend this thesis to other institutions or individuals for the purpose of scholarly research.

Nathan C. Brown

I further authorize Princeton University to reproduce this thesis by photocopying or by other means, in total or in part, at the request of other institutions or individuals for the purpose of scholarly research.

Nathan C. Brown

Abstract

Due to its intrinsic properties conducive to the synergy between form, use, and sustainability, the hyperbolic paraboloid (hypar) shell is a viable solution for a diverse range of structural applications. The hypar shell building boom has been over for thirty years and often relied on a cost relationship between labor and materials that no longer exists. However, with advances in materials science, construction techniques, and numerical modeling, shells can once again become viable structures given the contemporary context of sustainable building. The geometric properties of the hypar shape allow for a seemingly limitless potential for innovative structures that behave intuitively and can be formed using straight lines. The inherent versatility, structural efficiency, economy, and constructability of hypar shells are all features which make the form and its manifestation in concrete worthy of further research.

In order to understand what has been accomplished in the narrow field of shell building, a historical background on the development of shells is given, tracing the use of the hypar form and its precursors through the lens of a few key designers. Existing hypar shells are categorized based on form and evaluated in terms of efficiency, use, sustainability, and other performance indicators. The categorization of the hypar form discusses prominent design trends and functions as a field of reference for the spatial and programmatic possibilities of hypar shells, able to both inform and inspire future designs. In addition to the generalized study of the hypar form, the Miami Marine Stadium is used as a case study for the successful implementation of a hypar shell structure in an urban context. Study of the stadium's hybrid hypar umbrella/folded plate design includes a discussion of the construction narrative, a finite element structural analysis, and an evaluation of the stadium's sustainability and importance to the city of Miami.

Last, this thesis proposes a conceptual design for a Latin American hypar roof structure covering an exercise area in response to the regional environmental conditions and widespread health risks associated with inactivity. This design takes into consideration the interplay between form, use, and sustainability, and aims to be holistic while considering the structural, environmental, economic, and cultural forces at work in the design of any structure.

Acknowledgements

I would first like to thank my advisor, Professor Sigrid Adriaenssens, for all of her guidance, energy, and enthusiasm while working with me on this project. It was Professor Adriaenssens who first introduced me to the Miami Marine Stadium, consequently giving me the idea for the entire thesis. Without her time spent in discussions and giving me helpful feedback, this would not have been possible.

I would also like to thank a number of other graduates, professors, and students associated with the Civil and Environmental Engineering department. Ted Segal provided valuable insight into my design, and Ashley Thrall and Sylvester Black were helpful in advising me on how to complete various models. A thank you is also due to the Professors Garlock for their teaching, independent work help, and advising throughout my academic career. Along with the work of David Billington, Maria Garlock's previous research and documentation of Candela's shells was immensely valuable.

For help in better understanding the context and history of the Miami Marine Stadium, I must thank Don Worth, Hilario Candela, and others associated with Friends of Miami Marine Stadium and the Biscayne Bay Yacht Club. Not only did their correspondence aid my research, but their hospitality and friendliness made my trip to Miami a wonderful experience, graciously offering me a window in their community for a short time. I am especially grateful to Jack Meyer and his family for allowing me to be a part of the event in his honor, and to Jack specifically for the time he spent teaching me about his stadium and serving up many personal stories and words of wisdom.

To the people who offered administrative, technical, and other forms of support while writing my thesis, I am sincerely grateful. Tiffany Sirois, Jane Soohoo, Islam Elnaggar, and Minmin Fan were helpful in taking care of details associated with my research. I am also thankful to Lauren Crandell for running the thesis workshop, and to the School of Engineering and Applied Science for their generous financial support of my work.

Last, thank you to my family and friends for offering love, support, and encouragement throughout this process. My years at Princeton have reinforced how important you are to me, and have taught me what to strive for in my daily being: to serve God, love others, and make my family proud.

Table of Contents

List of Figures

** All images and figures are generated by the author unless a credit is given in the caption.*

1. Vision and Objectives

In the second half of the twentieth century, structural designers began to noticeably push the limits of both the material properties of reinforced concrete and gravity itself through their innovative designs in thin shell structures. Although at first these structures seemed radical and were not accepted by the established structural engineering community, their performance and efficiency in a wide variety of shapes and locations provided enough evidence to convince engineers of their effectiveness in solving structural and programmatic design challenges such as stadiums, factories, chapels, and other large span structures. This confidence lead to an extensive shell building movement, which was stimulated by a few key designers working throughout Europe, the United States, and Latin America. These 'master builders' practiced mostly in the 1950s and 1960s, leaving structural engineers of today with a wealth of examples from which to learn about the design and behavior of thin shell structures (Beles *et. al.,* 1976).

The main advantage inherent in concrete shells is their ability to span large distances, often unencumbered by columns or other interior obstructions, while minimizing the use of materials. Although formwork for shell construction can be expensive, a reduction in materials contributes to an overall reduction in cost, which can combine with the aesthetic value of a soaring, impossibly thin roof to make for an attractive design alternative to more conventional construction techniques. Thin shells were also able to take advantage of the relatively lower labor costs in many countries throughout the 1950s and 1960s, which is one reason why they were much more popular then than they are now. Despite innovations in thin shell construction techniques, such as digitally fabricated or inflatable formwork, using straight lines to generate the curved forms of ruled surfaces, and modular construction, thin shell building remains a labor intensive practice, primarily because rebar is difficult to place and concrete must be poured and shaped with exceptional care and precision. In previous decades, labor was relatively cheap and materials were more expensive, creating a perfect context for the spread of shells. Today, this relationship between labor and material costs no longer remains (Adriaenssens *et. al.*, 2012).

However, with sustainable building practices becoming arguably the most important focus of the construction industry today, shells may once again become a viable option for contemporary structural designers (Draper, 2008). The efficient use of concrete can drastically reduce the indirect carbon footprint of the construction process by cutting down on carbon emissions released through the production of concrete. Academia has noticed the possibilities allowed by shells, and progress is continually being made in developing more advanced construction techniques to reduce labor costs and again make shells an attractive bottom-line design alternative (Meyer *et. al.*, 2005). In light of these developments, it is of vital importance to continue studying shells and their applications to understand what forms are conducive to good structural design. An understanding of form, rather than the scientific principles that dictate its behavior, is a main goal of this thesis, and this aim is shared by many structural engineering researchers and practitioners. In the words of Professor David Billington, "science, by which we here mean primarily classical physics, seeks only to understand how given forms behave, whereas engineering design must first choose a form and it is for that activity that historical knowledge is required" (Billington, 1982, p. 1).

One of the most versatile and useful forms that can be employed in thin concrete shells is the hyperbolic paraboloid (hypar), often referred to as a saddle or umbrella depending on its manifestation. A hypar is a doubly ruled shape defined by the combination sets of two opposite-facing parabolas perpendicular to one another. Because it is doubly ruled, every point on the curved surface of a hypar lies on two separate straight lines. This property makes the hypar useful to designers in two different ways: it allows for easy visualization of construction, and it makes structural analysis of a shell intuitive and analogous to simpler forms, even if advanced numerical methods are not available (Billington, 1982). Beyond these simplifying properties, however, lies the form's versatility, which is the main reason why imaginative structural designers such as Félix Candela were drawn to the form. A hypar can be placed in an array, or it can stand on its own, with varying degrees of symmetry or asymmetry. The geometry can be inverted and fashioned into an umbrella, or it can be turned on its side to form a complex barrel vault. Its edges can be cut to match the traditional footprint of a building, or they can be left free to express the true thinness of the structure while creating an irregular yet rational plan. In other words, there is an unlimited potential for form within the basic geometry of the shape. Yet, each new way to employ the hypar retains the inherently intuitive behavior and potential for structural efficiency and economy of materials when utilized in the design of a thin shell structure.

This thesis aims to explore these structural, functional, and environmental aspects of the hyperbolic paraboloid form as employed in thin shell structures. Throughout each chapter, hypar shells are viewed in the context of a broad conception of sustainability, renewing the societal relevance of thin shell structures. In order to fully understand what has been done before in this narrow field, Chapter 2 gives a historical background on the development of thin shell structures and traces the use of the hypar form and its precursors through the lens of a few key designers. Based on examples associated with this historical narrative, Chapter 3 categorizes a diverse range of hypar applications according to configuration and evaluates them based on efficiency and sustainability performance indicators such as area and volume per amount of material. The categorization of the hypar form functions as a field of reference for the spatial and programmatic possibilities of hypar thin shells, able to both inform and inspire future designs. Given a drive towards the future, performance evaluations take into account the current context of sustainable design, which is discussed in detail to give a foundational understanding for determining the feasibility of hypar shells as a modern, sustainable design solution.

In addition to the broad study of the doubly curved form, this thesis seeks to add breadth to the existing knowledge and scholarship on the hypar shell through an in-depth analysis of one particular structure. Many of the hypar structures by famous designers such Eduardo Torroja and Félix Candela have already been analyzed in detail (Garlock *et. al.*, 2008), but there are few analyses on these types of forms in the United States. This is especially true for designers who employed a variety of different forms throughout their careers instead of specializing in thin shells, and thus do not fit into the canon of recognized structural artists. For example, extensive research has been completed about the structures of Jack Christiansen in America (Segal, 2008), but much less has been done to understand the work of less famous designers.

For this reason, the Miami Marine Stadium, designed by architect Hilario Candela and engineer Jack Meyer, is used in this thesis as a case study. The stadium's design narrative is provided in Chapter 5, establishing the design as a compromise between a visionary architect and a pragmatic engineer who combined efforts to build a successful hypar shell structure in an urban context. The concrete modernist stadium is significant not only for its beautiful visual form and complex geometry, but also for its structural and environmental performance (Adriaenssens *et. al.*, 2012). Chapter 6 uses the tool of Finite Element Analysis (FEA) to give a structural analysis of the form of the stadium roof. These calculations show the structural efficiency of the folded hypar form generated by the designers, as well as how this allowed for economy in construction materials. Although the stadium has many different components made from concrete, this thesis focuses on the combination folded plate, hypar umbrella form of the roof, since this is the most exceptional structural aspect of the design. To gain a more thorough understanding of the efficiency of the structure, the original design is also compared with FEA models that contain modifications, such a removing the thickening groins and stiffening diaphragm. The comparisons highlight ways in which the structure was already optimized or could have been improved. These modifications were chosen to determine the behavior of the roof had it been designed as a pure hypar with no folded plate action, giving insight into a view of the hypar as an inherently efficient form.

Last, Chapter 7 proposes an integrated structural design application based on the findings of this research on the hypar form. The application, a roof covering for an activity center in Latin American regions that require shelter from both large volumes of rain and near-equatorial sun, includes renderings, basic analysis, and a discussion of environmental and social sustainability of the structure. In addition to being shaped by historical and experimental research on form, the hypar roof's design is influenced by considerations of the environmental, material, economic, and social aspects of such a structure. The design is the culmination of everything learned throughout the process of researching for this thesis, and illustrates how the structural form of the hypar can be used in the modern developing world to respond to a particular social problem and subsequent programmatic requirements. The scope of this design restricts itself by remaining in the conceptual phase and focusing on preliminary structural design, leaving details concerning alternative load cases, foundations, and construction techniques for further research.

Throughout the historical, analytical, and design sections contained in these chapters, this thesis seeks to highlight the relationship between three broad concepts related to the construction of hypar shells. The first concept is that of form, which includes geometric versatility and structural efficiency. The second concept, use, refers to the architectural programs contained within hypar shells, as well the functional aspects of a structure including the control of light, air, and energy. The last concept is sustainability, which is a function of the environmental, economic, and social components of a structure's given context. These concepts are interrelated, and interplay between the three creates the design narrative of any piece of the built environment. In light of this reality, the hypar shell is a viable solution to many structural applications, because it has intrinsic properties conducive to the synergy between form, use, and sustainability that is necessary for a shell design to be successful.

2. Literature Review

2.1. Historical Developments in Thin Shell Structures

Thin concrete shells represent a substantial step forward in the art of building after forms such as masonry construction, lattice systems, and rigid-joint frames had already been established by structural engineers in the nineteenth century. One of the earliest examples of a thin concrete shell was a Romanian mosque built in 1905 by engineer Gogu Constantinescu, which had a span of 10 m and a thickness of only 5 cm (Beles *et. al.*, 1976). Although a few other shells were constructed over the next ten years, it was primarily post-World War I industrial development and the publication of the first simplified theories for the calculation of shell roofs that were responsible for an upsurge in new engineering forms throughout the 1920s and 1930s (Beles *et. al.*, 1976). Shell design continued to advance in both structural efficiency and popularity for the next thirty to forty years, after which the popularity of shells began to decline. Throughout this steady development, a variety of methods arose for the design and construction of such shells, with many techniques following the emphasis of a specific national tradition. Together, these separate traditions form a large pool of examples from which to extract technical knowledge and design inspiration in the field of concrete shell engineering.

In order to discuss only the examples of historical thin shells most relevant to the study of the hypar form and the Miami Marine Stadium, the scope of this review is restricted in a number of ways. First, designers are primarily discussed in terms of how they relate to shell design in North America, although it is often necessary to touch on Europe to describe how ideas developed and eventually arrived in the United States or Mexico. Second, this study ignores the many advances made in shell design through methods of physical analogy. A number of famous shell designers, most notably Heinz Isler, designed and built many shells relying on such techniques as the creation of plaster-covered hanging cloth membranes (Garlock *et. al.*, 2008). Although these structures are innovative and interesting in their own right, they are primarily irrational forms and do not share certain construction techniques and aspects of structural behavior with geometric shells, and thus are not directly relevant to this study (Garlock *et. al.*, 2008). Last, a distinction is also made between shells in which the design driver is purely sculptural and those in which the drive is structural. In the context of form, use, and sustainability as defined by material and economic efficiency, structural shells relate to the study more directly.

2.1.1.Dischinger, Finsterwalder, and the German School

In the early stages of thin shell development, the major advances came out of the German tradition of mathematical calculations. Franz Dischinger, a German engineer, began looking at thin shell hemispherical domes while working for the building firm of Dyckerhoff and Widmann A.G. (Billington, 1983). In 1922, Dischinger and a colleague took out a patent for the Zeiss-Dywidag system of thin shell concrete roof structures, but being used to the German drive for mathematical analysis, "they felt uncomfortable with any structural form that they could not analyze mathematically, and hence immediately set out to find a mathematical formulation for domes" (Billington 1983, p. 173). Out of his theoretical efforts, Dischinger was able to publish a full mathematical treatise on the structural behavior of thin shell domes. This paper included a number of physical images of dome structures that were derived directly from mathematical formulas (Billington, 1983). Although Dischinger left many complications unsolved and was self-limited by his strict adherence to shapes that he could understand mathematically, he nevertheless set an important precedent for reliance on geometry in shell building. The tradition of using mathematics and geometry to generate structural forms eventually moved from circles and rectangles to include hypars and other more complicated geometry.

A slightly younger colleague of Dischinger's who worked at the same design firm would push both the theory behind thin shells and their built manifestations to another level. Moving from circular domes to the linearly extruded barrel shell, Ulrich Finsterwalder first published formulas supporting barrel shell design in 1933. His theory, which is technically called the membrane theory and has subsequently been used extensively in shell analysis, ignores any shell resistance to bending and was used by Finsterwalder to prove that his barrel shells required tangential supports (Billington, 1983). Each of his solutions was restricted to circular barrels, the only shape he was able to fully analyze using mathematics; later designers would use shapes that eliminated the need for additional tangential support. Despite the limitations of Finsterwalder's theory, he relied on it to build many structures, and he was still able to prove that shells were economically competitive and can be substantially lighter than other types of roofs (Billington, 1983). The built work of Finsterwalder was thus foundational, since both the economy and lightness of shells are fundamental pieces of the argument for the shell as a viable alternative in various design applications. In conjunction with Dischinger and other German designers, Finsterwalder is credited with providing ample evidence for the claims of economy and efficiency, as well as establishing the geometric approach to shell design.

2.1.2. Anton Tedesko in the United States

Although Anton Tedesko worked with Dischinger and Finsterwalder in Germany early in his career, his design style does not fit neatly into the German school. This is partially because he is notable for both his introduction of shells into the United States as well as his focus on constructability during the design phase of the project, a trait shared by designers in the Spanish tradition. The influence and design principles of Tedesko are thus complex in how they relate to other schools of shell building, since he gained inspiration and knowledge from different traditions and had a profound impact on many later designers. Chronologically, Tedesko practiced in the years directly following the successes of his fellow German shell builders. Thus, for the sake of continuity, he will be discussed before the other design schools and later shell designers, and also be given his own section as a testament to his stature and singular design accomplishments.

Tedesko's main unparalleled accomplishment was the establishment of shell building in the United States, for which he alone is credited. According to the research of David Billington and Maria Garlock, "rarely can historians attribute to one person the introduction into society of a new and useful engineering idea. We have no difficulty, however, in attributing to one structural engineer, Anton Tedesko, the introduction of thin shell concrete-roof structures into the United States" (Garlock *et. al.,* 2008, p. 30). In early 1932, Tedesko was sent by Dycherhoff and

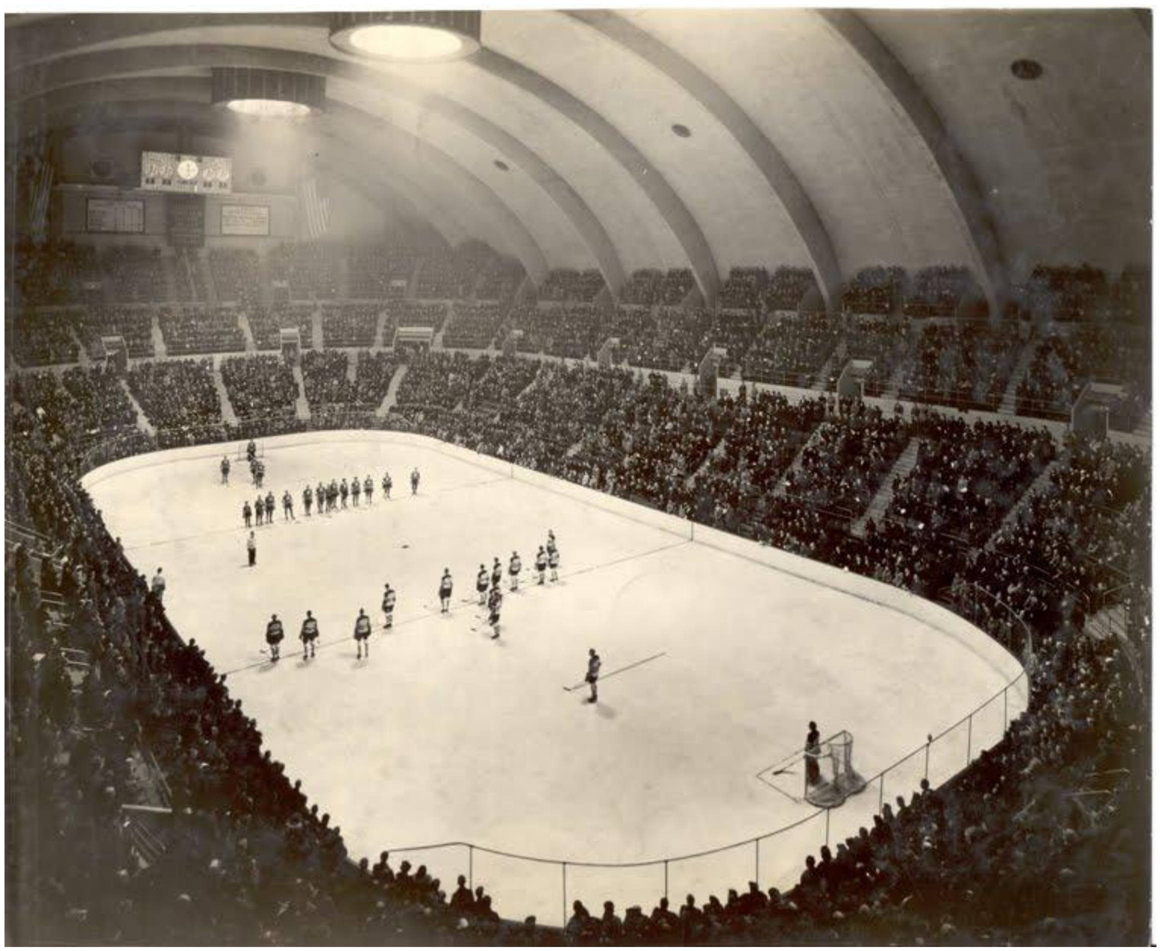


Figure 2.1: The interior of Tedesko's Hershey Arena. Photo courtesy of Nanette Clark.

Widmann to the United States in order to expand the firm's international operations. Four years later, Tedesko built the first large thin shell structure in the United States, a hockey arena in Hershey, Pennsylvania (See Figure 2.1). The arena was impressive both for its size (70.7 meter span, 103.7 meters in length), as well as the low cost and efficiency of construction Tedesko was able to achieve (Garlock *et. al.*, 2008). The success of the Hershey design and the needs of a relatively new air transportation industry allowed Tedesko to apply his experience in large span shells to the construction of airplane hangars, such as the one at Ellsworth Air Force Base (See Figure 2.2). Many of these designs included ribbing to support the shell, and this ribbing would only later be done away with by more risk-taking shell designers such as Félix Candela. Candela

Figure 2.2: Tedesko's hanger at Ellsworth Air Force Base, showing exterior stiffening ribs. Photo courtesy of Nanette Clark.

even referenced Tedesko's ribs directly, designing his Bacardi Rum Factor as a critique of the intersecting barrel shells of Tedesko's Lambert Field Terminal Building. Candela would show that the bulky stiffeners of Tedesko's design are not required for a vault of that size as long the geometry is slightly modified (Garlock *et. al*., 2008). Nevertheless, shell designers in the United States are indebted to the built work of Anton Tedesko for his pioneering structures that stood unmatched in scale and lightness for some time. It is also worth noting that Tedesko built one of the earlier hyperbolic paraboloid thin shells in the United States, a gabled hypar roof in Denver in 1958 (Garlock *et. al.,* 2008). By this point, however, other designers had already been building concrete shells in the shape of a hypar, and thus he cannot be credited with bringing this innovation to America.

In addition to the contributions of Tedesko's theoretical and built work to the field of shell building, his ideas concerning the design and construction process have also been influential. Before joining the practice of Dycherhoff and Widmann, Tedesko spent a year in a practice that strongly emphasized design and construction within the same company, a principle that stuck with Tedesko and proved to be well-suited to the efficient construction of concrete shells (Garlock *et. al.,* 2008). As an example, Tedesko personally supervised the construction of the Hershey Arena, as well as designing it (Garlock *et. al*., 2008). Although the practices of architecture and engineering have changed substantially since the time of Tedesko and the tradition of the designer-builder may no longer be possible, acknowledgement of the tradition is of vital importance in understanding the history of thin shells. The designer-builder tradition is shared by members of other national styles as well, and will be discussed throughout the rest of this chapter.

Despite changes in the industry, some of Tedesko's experiences can still be useful for designers today. Tedesko spent much of his career collaborating with architects, and his ideas on how the joint design process should be executed are relevant regardless of era. Once, when Tedesko worked on his gabled hypar roof in Denver with I.M. Pei, he felt compelled to balance the demands of the architect with the demands of physics (Garlock *et. al.,* 2008). For the Denver roof, Pei wanted a shell completely without ribs, unlike many of Tedesko's previous designs. In order to accomplish this, Tedesko knew he would have to defend the structure from buckling, which lead him to add two intersecting slab bands to reinforce the flat middle of the roof.

Figure 2.3: Courthouse Square Extension Roof by Tedesko, later called the May-Daniels & Fisher Department Store Entrance Hall before it was demolished in 1996. Photo courtesy of Nanette Clark.

(Garlock *et. al.,* 2008). The Denver roof example, as well as the Miami Marine Stadium discussed later, proves that architects and engineers can indeed collaborate to produce efficient, cost-effective shells. Throughout this joint design process, Tedesko outlines three principles on which the success of shells depends:

1. Designers learn from having experience with full-scale structures.
2. Engineers should visualize the actual construction during the design stage; the economy of shells depends on the close collaboration of designer and builder.
3. Even in working with prestigious architects, the engineer must stand firm, making it clear as to what can be done and what should not be done. (Tedesko, 1980)

All three principles can still be applied to the design of shells, and the experience of Tedesko gained over decades of shell building has been relied upon by many designers as they pushed the narrow field of shell building forward.

2.1.3. Pierre Luigi Nervi and the Italian School

Pierre Luigi Nervi, an Italian shell designer who practiced almost exclusively in Italy between 1930 and 1960, was notable in his contribution to shell construction for three reasons: he focused on aesthetics; he was able to win cost competitions; and he insisted that a structure must understand and fit into its local context (Garlock *et. al*., 2008). In considering himself an artist tasked with creating beautiful structures, Nervi inherited the great Italian tradition for visual aesthetics from the architects of both antiquity and the Renaissance, and it was often to these designers he looked for inspiration. While German designers were publishing technical treatises on the behavior of shells and domes, Nervi was instead writing papers about the art of building, past architectural achievements, and the new aesthetic of modern technology (Billington, 1983). Nervi chose to analyze ancient Italian monuments and use an intuitive understanding of the flow of forces to apply their aesthetic to a new, modern material in an efficient and cost-effective manner. The influence of Italian aesthetics can clearly be seen in comparisons between ancient

Figure 2.4: A comparison of the Pantheon roof to Nervi's Little Sports Palace. Although the Pantheon contains radial coffering and the Palace contains meridional ribs, the lightening effect is the same. Left photo courtesy of the blog l*ejournaldelouise*, right photo courtesy of the blog *extremegroundhopping*.

or Renaissance structures and the masterpieces of Nervi. For example, the coffering technique used to lighten the dome's shell in the Pantheon and the Basilica of Constantine is similar to Nervi's later design of the Little Sports Palace (See Figure 2.4). One of Nervi's newer, more advanced forms of coffering is exhibited by the roof of Nervi's Large Sports Palace (See Figure 2.5), in which material is taken out of the cross section of transverse ribs rather than the roof itself. The two great domes of the Renaissance, St. Peter's and Brunelleschi's dome in Florence instead contain meridional ribbing, a technique which Nervi borrowed for the little sports palace before he developed his own method of vaulting (Billington, 1983).

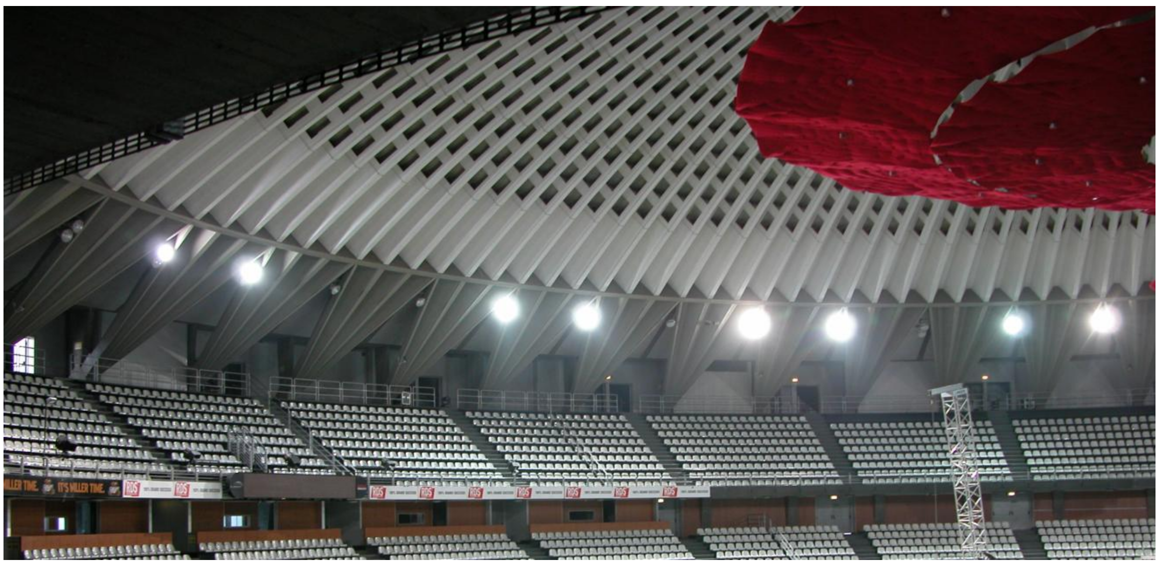


Figure 2.5: Nervi's Large Sports Palace in Rome, showing a coffering technique in which material is taken out of the cross section of radial ribs. Photo courtesty of Terri Boake.

Although Nervi's aesthetic sense—often taken at the expense of analytical calculations—may seem out of place in the engineering field, it is indeed what made him exceptional, and was often his greatest asset. The attractiveness of his structures inspired not only other shell designers, but also the architects with whom he collaborated and the general public who experienced his work. Félix Candela, arguably the greatest hypar shell designer of all time, wrote: "What was, then, the secret of success in Nervi's work? For me, this secret lies simply in its beauty; a beauty capable of being understood and appreciated, not only by the professionals, but also by the laymen" (Candela, 1964, p. 15). Without this ability to appeal to many different groups of people, Nervi would not have enjoyed as much success in shell building, and this can be a lesson to any designer working with highly visible structures. Nervi's aesthetic abilities must also be understood in conjunction with the efficiency and economy of his structures. He was not a designer who required hugely expensive materials and structural systems to achieve beauty, but rather broke into the field due to his ability to outbid competitors in terms of direct cost (Garlock *et. al.,* 2008). The combination of these two factors is the legacy of Nervi, who proved that beautiful structures can be achieved with economy, a notion fundamental to the spread of concrete shells.

Nervi's assertions about how the local context of a structure must influence its design are also significant, and may be worth revisiting given the focuses of the current design community. Nervi once wrote: "many times I have refused to accept commissions…in countries with whose possibilities for building [large structures] I was not familiar in order to avoid running the risk of designing shapes and structures which might prove impossible to build" (Nervi 1965, p. 24). Nervi's sentiment can be true on a number of different levels. A structure may be impossible to build because it does not fit the cultural background of the community in which it is proposed, and thus it would not be accepted in the first place. Nervi may also have been referring to the codes and building industry in a particular area. Certain building techniques may not be practiced or accepted in a given region, proving another roadblock to the construction of a large building. However, the issue of locality must be understood today in terms of energy and sustainability as well. The current housing crisis in Nigeria can serve as a contemporary example of how problems occur when the locality of a building site is not taken into account. Part of the reason more houses are not being built in Nigeria is due to the incredibly high cost of building materials in the country today (Ademiluyi, 2010). Yet, the crisis is worsening partially due to the prevailing notion that development is signified by the construction of traditional (often Western) buildings. A desire for Western forms has prevented the government from looking into cheaper material alternatives that may fit the local industry better, such as timber (Prier, 2011). Thus, Nervi's adamancy that the local context of a site must influence its design is a considerable contribution to the field, in addition to his aesthetic and economic successes.

2.1.4.Eduardo Torroja and the Spanish School

Eduardo Torroja, who can be called the father of the Spanish school of shell building, was able to search for smooth, ribless surfaces that expressed their true thinness out of the Spanish tradition of Catalan vaulting (Billington, 1983). Although Torroja was dependent on the previous work of Antonio Gaudi for inspiration, Gaudi worked primarily with masonry, and as such Torroja was the first Spaniard to work extensively with reinforced concrete. Torroja designed a variety of shells in Spain beginning in the 1930s, experimenting with double curvature and free edges in addition to traditional domes and barrels. Not coincidentally given his progressive design ideas and influence, Torroja's first major work bears a striking resemblance to the Miami Marine Stadium, although it was completed nearly thirty years earlier (See Figure 2.6).

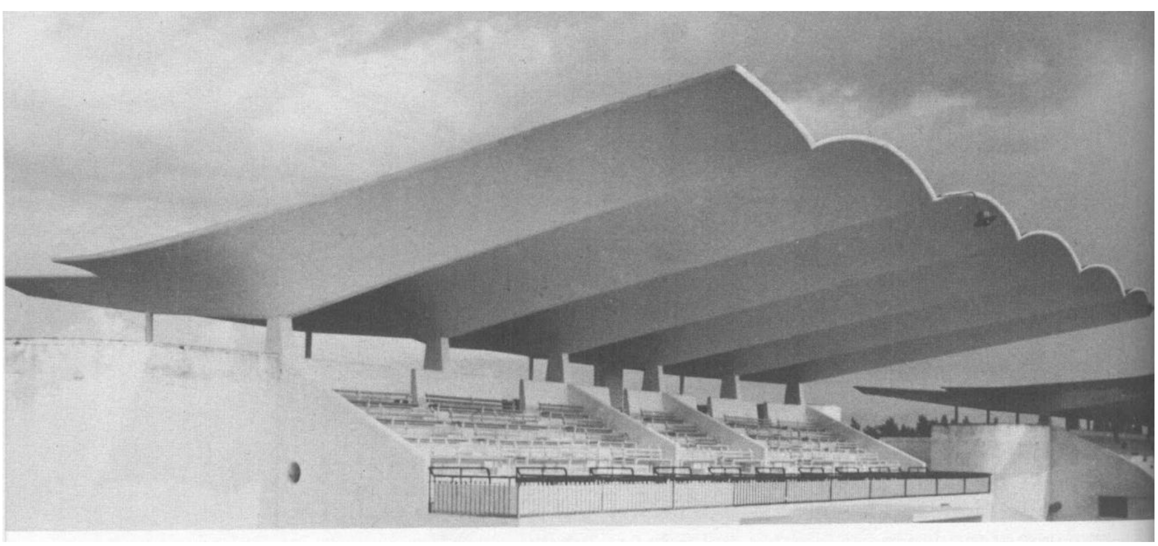


Figure 2.6: The cantilevered thin shell roof of Torroja's Zarzuela Hippodrome. Photo courtesy of *VAUMM*.

The Zarzuela Hippodrome roof, which is a cantilevered hyperboloid shell, was built in Madrid in 1935 (Billington, 1983). The roof is considerably smaller than in the Miami Marine Stadium (the cantilever is only 12.8 m), but it contains no stiffening ribs and exhibits an astonishing thinness ranging from 15 cm at the support to only 5 cm at the edge (Billington, 1983). The hippodrome roof had to withstand a catastrophic event when it was bombed during the Spanish Civil War, but it survived and remained standing as a testament to the strength afforded the shell by its curved form. When asked what inspired the design, Torroja responded that the shape was not purely rational or purely imaginative, but that both aspects of design combined to synthesize a structure that resulted from a combined logical and inventive process (Billington, 1983).

Although the Zarzuela Hippodrome was Torroja's most acclaimed design employing double curvature, he also conceived of structures that were barrels and domes, much like his German and Italian counterparts. However, these shells were distinct in that they did not include ribbing, leaving a smooth, plastic aesthetic (Billington, 1983). Two examples of this smoothness are the Frontón Recoletos, a barrel shell destroyed in the Spanish Civil War, and the Algeciras Market Hall, a large dome that used horizontal ties rather than exterior buttresses to counteract horizontal spreading (Billington, 1983). The thinness and plasticity of Torroja's shells inspired a

Figure 2.7: The Frontón Recoletos on the left, and the Algeciras Market Hall on the right, both by Torroja. Left photo courtesy of *Technomadas*, right photo courtesy of *archINFORM*.

number of Spanish designers, most notably Félix Candela. Torroja also passed on the legacy of combining geometry, mathematical formulas, and a form-finding approach to Candela, who continued pushing the field of shell design forward. Since Candela was a master of the hypar form and is thus highly specific to this thesis, he is discussed in his own section.

2.1.5. Félix Candela: Master of the Hypar Form

Following in the Spanish tradition after Torroja, Candela's interest in shells began when he was a student at the *Escuola Superior de Arquitectura* in Madrid, from which he graduated in 1935 (Billington, 1983). After graduating from school, Candela received a travelling fellowship to study the shells of Finsterwalder and Dischinger in Germany. Unfortunately, the outbreak of the Spanish Civil War prevented him from pursuing this fellowship, and he began fighting with the republican army against Franco. When Franco won, Candela fled to France, from where he was able to make his way to Mexico City after a brief internment in a concentration camp (Billington, 1983). In Mexico, Candela established himself over the course of a decade as both an imaginative designer and an efficient builder. Candela preferred to rely on experience at full scale in his designs, and he became more daring with each structure. His highly successful business, which generated a considerable amount of revenue from umbrella-based roofs, supplemented his income and reputation, allowing him the freedom to play with the hypar form. Candela used opportunities gained by his stature to build structures that used previously unexplored forms and were visually and structurally advanced compared to earlier shells.

According to Professor Billington, "Candela broke the stranglehold of academic science on thin shells and showed how beauty and utility could combine and open up limitless new possibilities for form" (Billington 1983, p. 190). This is not to say that Candela was unskilled in mathematics, since he was a trained architect who taught himself shell theory and even published a number of papers on membrane theory in shells (Billington, 1983). However, his designs are decisively more imaginative and varied than the shells that were built before him. This is largely due to his feeling of liberation from the academic discipline, which he gained gradually through experimentation and an understanding of full-scale structures (Garlock *et. al.*, 2008).

Given his desire to explore the boundaries of thinness and aesthetic performance while relying on the relatively manageable membrane theory, Candela was well suited to the hyperbolic paraboloid. Out of this curved shape, he was able to build a variety of structures, which took many different configurations and functions. Candela's hypar structures are discussed in detail as part of the classification system for the hypar form, but in general they shared a number of common characteristics. Most notably, Candela's structures carried the Spanish tradition of smooth, plastic structures with free edges and clear structural expression to its most advanced state (Billington, 1983). With the exception of his earlier work, Candela's shells require minimal ribbing. He often inverts the hypar shape into an umbrella, and this inversion forms the basis for many of his designs. As Candela was a builder as well as a designer, he also built tessellations into many of his structures, which allowed him to reuse formwork and achieve remarkable economy (Billington, 1983). Overall, Candela showed how an understanding of physical shell behavior and an imagination unbound by rigid dependence on technical theory can achieve beautiful, efficient, and cheap structures for many design applications. He is both the culmination of the previous developments in thin shell design, and also the most central figure for a feasibility study of hypar concrete shells.

2.1.6. Hilario Candela, Miami Marine Architect, and his Main Influences

Hilario Candela, a distant cousin to Félix, is a Cuban-born architect who was only 28 years old when his Miami Marine Stadium was completed in 1963 (*Friends of Miami Marine Stadium*). Since the construction of the stadium, Candela has been practicing extensively in the Miami area, although the stadium is his signature work in the field of concrete shells. After being educated at Georgia Tech, Candela returned only briefly to Cuba before the country's political situation caused him to move to the United States permanently in 1960. Candela's short time in Cuba exposed him to the "inner circle of Cuba's innovative experimenters with thin shell concrete and expressive roof lines" (Adriaenssens *et. al.,* 2012), whose ideas he brought to his practice in Florida. The varied architectural ideas surrounding Hilario Candela do not represent a single, definable strand in the field of shell building, partially due to the fact that his architectural career did not focus exclusively on shells. Hilario was also not interested in pursuing the engineering side of shell behavior, limiting his ability to design shells on his own. Since his influences and interests were so broad, many of them are not directly relevant to this feasibility study of thin shells. However, a discussion of his influences is included to give context to his Miami Marine Stadium, which is used later as a case study for the structural expression of a thin shell in the United States.

As an adherent to the architectural modernist tradition, Candela mentions a wide range of influences on his designs, from Le Corbusier and Mies Van der Rohe to his distant cousin Félix (Kleinman, 2010). The designs the stimulated Candela include a diversity of forms, such as earlier squared and block modernist shapes, the sculptural use of concrete by Latin designers like Oscar Niemeyer (See Figure 2.8), and the mature structural applications of thin shells by Torroja, Nervi, and Félix Candela. Given this variety, Hilario Candela's relationship with modernist design is complex: according to one architectural historian, "in the mature years of his career he could properly be classified as a member of the brutalist school, but with a twist. He always incorporated Caribbean design staples for controlling sunlight and capturing breezes" (Kleinman, 2010). Although some of Hilario's modernist architectural influences have little to do with shell building, they had the cumulative effect of instilling in Candela a love of concrete as a building material, which would influence him to experiment with the formal possibilities of expression allowed by concrete. When coupled with the creative and pragmatic ideas of a skilled engineer, Candela's desire to experiment with curved concrete eventually opened up the geometric possibilities of the Miami Marine Stadium.

Figure 2.8: Catedral de Brasilia by Oscar Niemeyer. Although this example was built in 1970, it exemplifies the use of structural concrete popular in Latin American architecture. Photo courtesy of Peterson Amadeu.

In addition to the variety of high modernist design examples digested by Candela, there were direct influences on the Cuban architect that dealt primarily with thin shells. The most important of these influences was Max Borges Jr., a Cuban architect for whom Candela worked in the summers during his years at Georgia Tech (Kleinman, 2010). Borges practiced in Cuba for nearly twenty years, leaving among his buildings two signature works: the Tropicana Nightclub

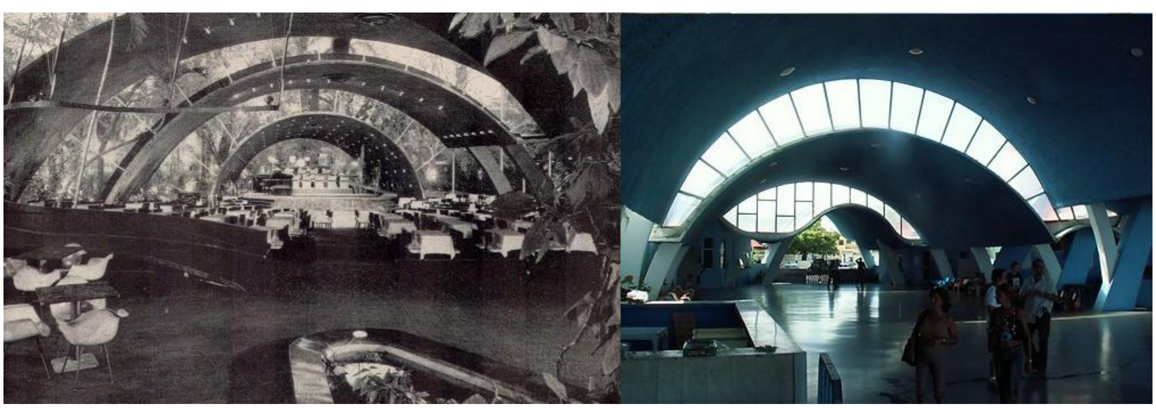


Figure 2.9: Borges' Nautical Club (left), and Tropicana Nightclub (right). Photos courtesy of John Pilling.

and his building for the Nautical Club (Rodriguez, 2000). Both of these structures employ curved, thin shell mismatched roofs that use gaps between different roof sections for natural lighting (See Figure 2.9). The Tropicana Club was built in 1952 to host a cabaret and casino, and it was "designed to make cabaret-goers feel as if there was nothing between them and the lush tropical night" (Antoniades, 2010). The architectural demands presented by the club exemplify a situation in which concrete shells are often at their best: light, open, airy structures in warm climates that need unobstructed space protected by rain and direct sunlight. The Nautical Club built by Borges is similar, although it is more squat and massive (Antoniades, 2010). Nevertheless, the Nautical Club achieves structural and architectural success in its own right, clearly communicating its function through architectonic expression and "establishing an almost literal analogy with the waves of the sea nearby" (Rodriguez, 2000).

While building these shells, Borges demonstrated the same care and precision as many of the earlier Spanish shell builders, proof of which is given by interviews with his colleagues and occasional anecdotes. For example, upon hearing a train coming nearby to his Tropicana Nightclub, a young Borges once scrambled to the top of his shell to measure vibrations from the train himself (Antoniades, 2010). However, Borges and others who pioneered thin shells as part of the Cuban modernist architectural tradition represent their own path in shell development, distinct from the other previously mentioned schools even as they are closely tied to Félix Candela in Mexico. Borges' structures must be understood as a landmark in the continuous development of thin shells and their appearance in the United States, but in a parallel when compared to the shells of Tedesko, because they came later and with a distinctly Latin architectural flavor. Borges may have been the most famous Cuban designer in thin shell concrete at the time, but he was part of a larger Latin movement that shared cultural design practices. In the words of Hilario Candela, Borges' work exemplifies "the expressive concrete architecture widely popular throughout Latin America and Brazil" (Candela, 2011). The smooth, beautiful curves of thin shells are well-suited to the climate, building industry, and culture of Latin America, which made the export of their design principles to the city of Miami well accepted.

2.1.7. Later Developments in Thin Shells

Although advancement in the field of thin concrete shells slowed considerably after the 1960s and early 1970s, it did not stop altogether. The last of the great shell designers of in the United States, Jack Christiansen, finished his most spectacular work in 1976 (Draper, 2008). Built in Seattle, Christiansen's Kingdome used hyperbolically curved concrete in between numerous stiffening ribs to create a dome spanning 660 feet, easily the largest thin shell dome in

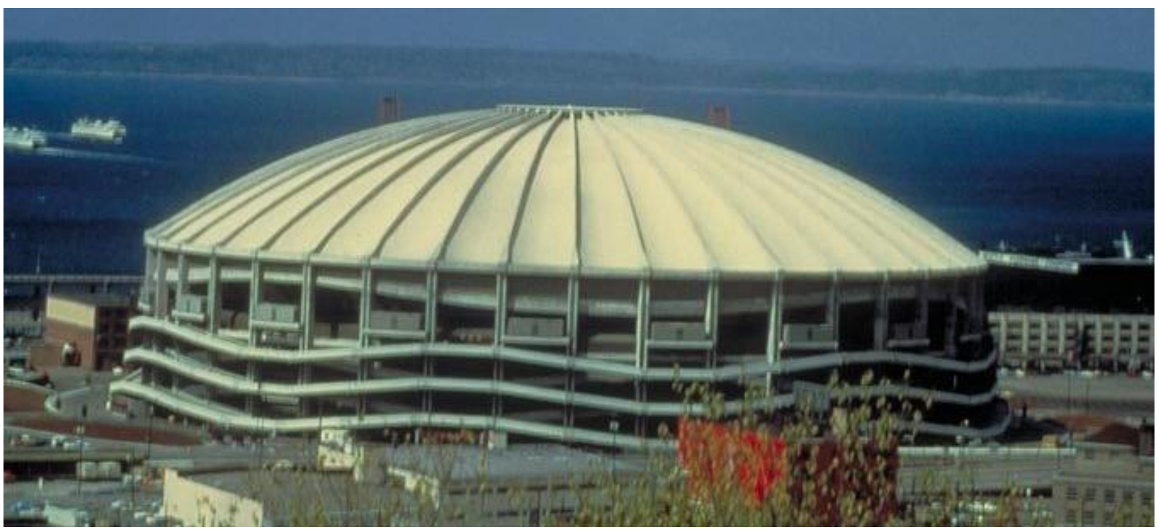


Figure 2.10: Seattle Kingdome by Christiansen. Photo courtesy of Judy Wood.

the world (See Figure 2.10) (MacIntosh, 2000). The structure was also praised for its economic efficiency since it was completed on budget at $30 million, making it the least expensive domed, air-conditioned building of its size (MacIntosh, 2000). However, with the possible exception of Heinz Isler in Switzerland, landmark thin shell structures such as the Kingdome largely stopped being constructed after the 1970s. This is due to a variety of factors that can broadly be summarized as a lack of trust in the continuing viability of shells and changes in architectural taste. Given this context, the focus of research in thin shells shifted towards the pioneering of construction techniques and materials science in an attempt to make shells viable from another angle (Meyer *et. al.,* 2005). Both the reasons for the drop in thin shell popularity, as well as their current viability, will be discussed extensively in the next section.

2.2. Current Viability of the Hypar Shell

To establish the importance of studying hypar shells, it is necessary to determine the usefulness of shell building knowledge to future applications. There were many favorable factors in the circumstances of each master designer-builder that were conducive to the construction of thin shell structures and the use of hypars in these designs. Unfortunately, many of these factors no longer exist, and the lack of interest in shells today is evidence of this reality. In his doctoral dissertation, former Princeton student Powell Draper provides a detailed explanation of the contemporary lack of interest, while also conducting an extensive evaluation of the current viability of thin shell structures. Draper isolates and addresses three main reasons for the lack of thin shell designs from today's engineers: their perceived high cost in relation to necessary labor, a falling out of stylistic favor with current architects, and the belief that they are too difficult to analyze to be worth an engineer's time (Draper 2008, p. 6).

With respect to perceptions of high cost, considerable work is being done to develop more sophisticated construction techniques that will reduce the amount of skilled labor and the price tag associated with this labor during construction. In particular, many advances come in the field of concrete materials science, such as the invention of stiffer fabric forms and fiber-reinforced composites (Meyer *et. al.,* 2005). These materials open the possibility for using inflatable formwork, which can drastically reduce costs compared to traditional scaffolding. Savings due to steel or glass fiber-reinforcing can also be realized directly in the construction phase by eliminating the need for placing reinforcing bars, which can be an expensive and tedious task. In addition, improvements in shotcreting technology have been cited as a way to reduce cost in shell construction. Shotcreting is a technology in which concrete is pneumatically projected through a hose onto a surface at high velocity, and using better materials to prevent rebound of the formwork during this process reduces application time and effort needed in cleaning (Meyer *et. al.,* 2005). In general, academia is also looking at methods of prefabrication and modular units in formwork, which could have an effect on the cost of shells as well.

The problem of difficulty in analysis can be partially addressed by advances in numerical methods. Although it is important to know the underlying principles of any computer program used for the purpose of design, Finite Element Analysis programs such as the one used in this thesis can greatly reduce the time and complexity of the design process. These tools have cut back on the need for closed-form analytical solutions in shell theory (Meyer *et. al.*, 2005), which can only be done by those with extensive backgrounds in mathematics. As more and more designers are being educated in the theory and use of these programs, the complex analysis required of concrete shells is becoming less of an obstacle to their construction.

The other way in which the complexity of structural analysis in shells can be reduced is by designing structures in the shape of the hypar, which lends itself to straightforward calculations through its geometrical nature. Candela was especially adamant about this second point, stating that “the hyperbolic paraboloid is the only compound surface that can be analyzed by simple statics. That is its real justification and a far more valid one than the beauty of its form” (Candela, 1958, p. 205). Although the general mathematical formulations involved in shell calculations lead to high order partial differential equations, the majority of hypar forms can simplified into one of two types. Most hypar forms behave either as a cantilevered beam or as a stiffened arch (Billington 1982, p. 25). These two main behaviors can be separated into additional categories, and although simple analogies must be validated through other methods, the structural behavior of the hypar is inherently intuitive. A more detailed, mathematical description of their behavior will follow later in this thesis. For now, it is simply important to know that the question of difficult calculations in shell design can be addressed in many cases by the simple geometry of the hyperbolic paraboloid.

Unfortunately, no technical advances will be able to eliminate the problem of style, as the field of engineering has a limited ability to change current architectural styles or trends. As the fields of architecture and engineering have diverged and each has become more regulated and specialized, the overall form of structures has become almost solely the domain of the architect. Today, engineers often work to simplify and analyze the increasingly irrational forms proposed by contemporary architects. Candela and Nervi did not have this problem; Candela was an architect and engineer who owned his own construction company (Garlock *et. al.*, 2008), while Nervi practiced as if all phases of design and construction should be under the guidance of one designer able to balance the scientific, technological, esthetic, and social factors in play. Nervi went even further, arguing that there should be no great divide between the education of the architect and that of the engineer, since “the existence of separate departments of civil engineering and architecture increases our uncertainties and clearly shows our lack of unified purpose. In fact, these departments do not differ so much in the subject matter they teach as in their viewpoints and the training methods they use, while both types of schools try to make designers and builders” (Nervi, 1956, p. 11). Nervi’s view on training and his treatment of construction were especially suited to the building of shells, which integrate the desires for economy, efficiency, and elegance inherent to structural design. Unfortunately for the use of shells, the influence of engineering on the overall form is no longer felt as strongly, and since architects primarily control the design, they are the ones who must somehow be convinced of the usefulness of thin shell structures.

In terms of making shells stylish again, it may be possible for the hypar shell form to take advantage of the current focus on sustainability in architectural design (See Figure 2.11). If shells are evaluated extensively in terms of their environmental impact and judged to be highly sustainable structures, this fact alone may be enough for shells to be given a second look by many architects and their clients, as long as sustainability is attached to lower cost. Unfortunately, many traditional means of measuring the sustainability of buildings do not apply to shells. It is thus necessary to provide a further explanation on current definitions of sustainability to determine how hypar-shaped thin shells might apply to each.

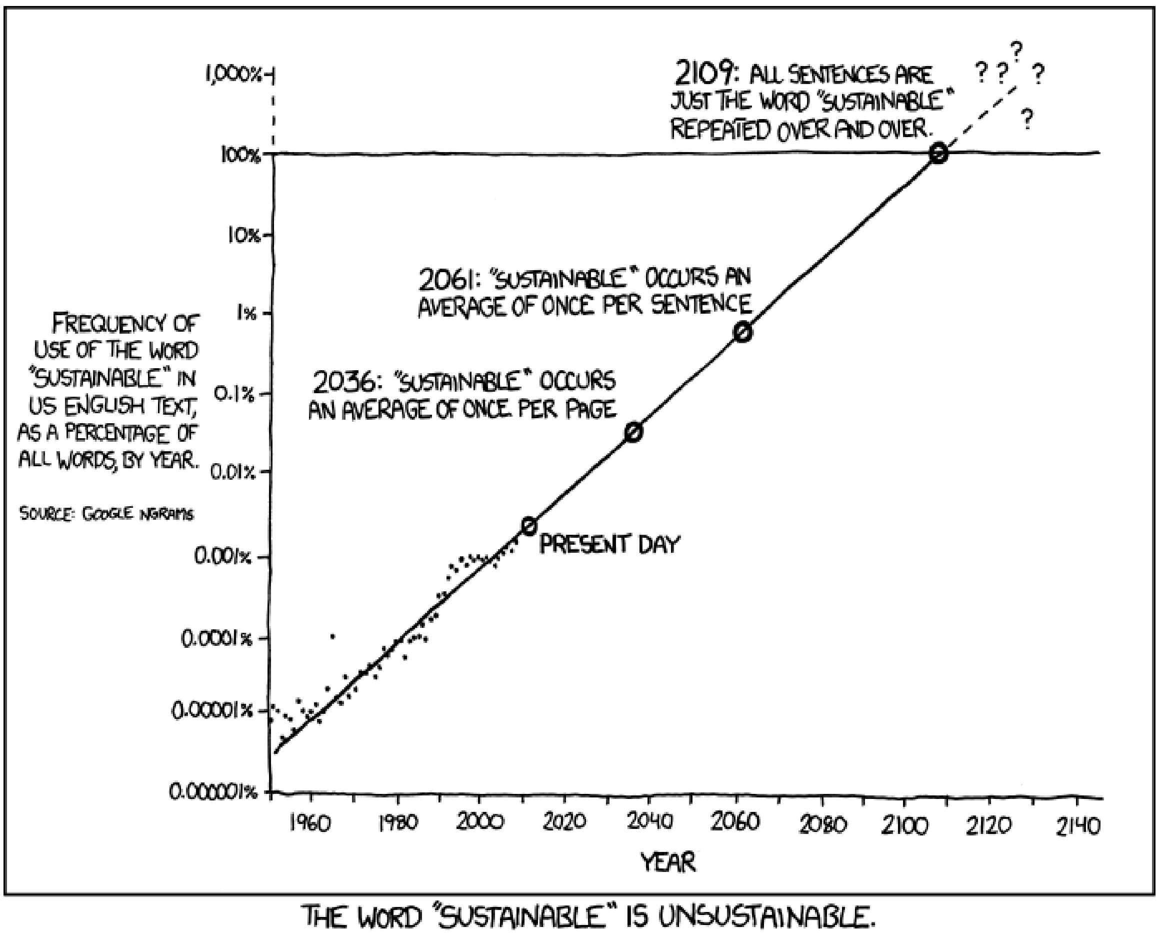


Figure 2.11: A popular webcomic, humorously illustrating the prevalence and popularity of 'sustainable' thinking in contemporary culture. Image courtesy of XKCD.com.

2.2.1. Sustainable Building

At its core, sustainability is a mode of thinking rather than a quantifiable property. The first attempt at defining sustainability in relation to development was made by the Bruntland Commission of the United Nations in 1987, stating that “sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs” (*U.N Documents,* 1987). Sustainable thinking is thus not about what builders should and should not build, but rather how they should approach the problems posed by human habitation. In every action that requires the consumption of resources, sustainability suggests that the least amount possible should be used, and every effort should be made to get the most out of the resources that are used (Borer, 2011). Although this definition remains broad, it can be effectively used as a first principle for which to understand more developed conceptions of sustainability.

Various attempts have been made at institutionalizing sustainability in the building industry, and in many cases a certain geographical area is governed by its own set of rules. The first instance of a concrete set of rules for sustainable building came from the *Conseil International du Batiment* (CIB) in 1994, which stated and prioritized Seven Principles of Sustainable Construction based on resource efficiency and ecological design. These principles dictate that builders reduce resource consumption, reuse resources, recycle, protect nature, eliminate toxics, apply life-cycle costing, and focus on quality (Kibert, 2005). The seven principles rely on considerations of structure, energy use, process and economics, and although structural engineers or architects can only control some of these considerations, the principles provide a general platform for understanding sustainable buildings. Further attempts to quantify sustainability are based on these same basic principles.

In the United States, the primary means for calculating sustainability in a building is the Leadership in Energy and Environmental Design (LEED) system of evaluation, which is administered by the U.S. Green Building Council. According to the newest version, LEED rating systems "evaluate environmental performance form a whole building perspective over a building's life cycle, providing a definitive standard for what constitutes a green building in design, construction, and operation" (LEED, 2009). There are many different LEED criteria related to various types of construction, but each one is organized into five different environmental categories: Sustainable Sites, Water Efficiency, Energy and Atmosphere, Materials and Resources, and Indoor Environmental Quality (LEED, 2009). Clearly, shell design only relates to a few of these categories. Thankfully, there are additional metrics available internationally for evaluating the sustainability of a building. One metric of particular importance, the Sustainable Building Assessment Tool© (SBAT), was developed by the Council for Scientific and Industrial Research in South Africa to evaluate the degree to which a building promotes holistic sustainable development (Gibberd, 2001). SBAT will be discussed in more detail in Chapter 7, but even it has many criteria for sustainability that do not directly relate to the design of a shell. Nevertheless, thin shell hypar construction embodies the basic principles of sustainability in terms of conserving resources and energy, and since the main evaluation system for sustainable building in the United States does not apply, other aspects of shells must be considered.

2.2.2. Sustainability in Shells

Since the usual metrics for determining sustainability in a building cannot be applied to the structural design of hypar shells, the industry must rely on the first principles of sustainability to point out the ways in which shells are sustainable: namely, that they are durable with minimal maintenance, and they reduce the amount of materials and resources needed when compared to conventional structures of comparable size and function (Draper, 2008). As an example of durability, many of Candela's famous structures in Mexico have been studied closely by recent engineering students. Despite their half-century age, exposure to the elements, and occasional earthquake loads, the shells tend to be in good condition with no need for repair, a testament to their durability (Draper, 2006). For evidence of shells' reduction in materials, one only needs to see the visual contrast in thickness between shells and other concrete forms. A comparison between shell and slab construction will be done qualitatively for many key examples of the hypar form in Chapter 4 of this thesis. It is worth noting that HVAC, lighting, and other types of energy performance during the life cycle of a building are measurable through modeling and precise measurement, but are outside the scope of a structural efficiency analysis. Although it is possible to insulate shells to reduce energy losses or leave holes in them to increase daylighting opportunities, this thesis will instead chose to focus on what aspects of sustainability can be controlled by intelligent, creative design of the overall form of a structure and measured accordingly.

2.3. Past and Present of Hypar Shells

To be successful in the field of shell building, a designer (or team of designers) must have a number of specific characteristics. He or she must have a thorough understanding of the structural behavior of thin shells, whether this understanding comes from theoretical roots or experience with concrete at full-scale. The ability to translate a complicated final geometry into an efficient, economic construction procedure is also a requirement. A keen aesthetic sense is helpful to a designer as well, since the success of a shell structure often depends on perceptions of its beauty to clients, building occupants, and the general public. Most importantly, a designer must have a daring, innovative spirit and be willing to experiment within the physical constraints of his or her building medium. This spirit ran through each of the different shell building schools regardless of a main emphasis on mathematics, cultural aesthetics, or construction techniques, and it is what drove the field forward to produce lighter, cheaper, and more efficient structures.

Although the period of explosive innovations in concrete shells is long over, the ability of reinforced concrete shells, especially in the hypar form, to solve a variety of engineering challenges should not be ignored in the contemporary design community. Due to changes in the building industry, architectural tastes, and the economic conditions of many developed nations, clients have been less willing to allow shell builders to experiment with new forms at full scale. Thus, many recent innovations have been restricted to advances in theory, materials science, or construction techniques. Nevertheless, precedents set by both the master shell builders of the pre-1970s and also by more recent researchers can inform the work of shell designers in the future. In light of the building industry's current focus on sustainable building practices, shells can once again be considered a viable design solution for many different structural challenges and geographic regions. In a search for efficient, sustainable, elegant concrete structures, a current designer can look to the past for inspiration while also driving the field forward with his or her own new ideas.

3. Technical Description of Hypar Shells

3.1. Introduction

Up until this point, hypar shells have only been discussed in general, nonmathematical terms. This section is meant to provide a more thorough understanding of the science, mathematics, and engineering components of shell structures. A description of the analytical and theoretical material used to examine shells lays a foundation for the study of various geometric configurations of hypars in the next chapter, and it also explains the primary methods used by engineers for designing shells before the application of advanced computer modeling techniques was made possible. This chapter begins with a generalized description of thin shell structures and curved shapes before moving on to a focused explanation of the hyperbolic paraboloid form. The focused explanation presents the mathematics behind the geometry and straight line generators of a hypar while also discussing a number of useful theories and analogies used by hypar designers. Many skilled engineers have proposed their own methods or theoretical frameworks for solving shells, and thus a full structural explanation of hypar shells would have to include geometry, numerous analytical and variational methods for obtaining solutions, and stress and strain states for both membrane and bending theory (Beles *et. al.,* 1976). However, if built in a "proper" form, a shell can be made to exhibit mostly membrane behavior and carry forces through tension and compression rather than bending (Draper, 2006). Furthermore, many master shell builders relied on only simplified theories or analogies to make calculations. Given the historical reliance on simplification and analogy, this thesis restricts the scope of technical background information on the hypar to only the relatively simple theories most widely used in the application of shell structures.

3.2. Reinforced Concrete Thin Shell Structures

Reinforced concrete shells are made by pouring wet concrete in place over steel rebar that has been strategically placed within the structure to take tension, bending, or other loads ill-suited to concrete. Formwork, usually made of wood, must be prepared beforehand in the shape of the eventual structure, and it is removed once the concrete hardens (Peurifoy *et. al.*, 2011). As with most reinforced structures, the steel is covered by a minimum amount of cover concrete, leaving a smooth, continuous finish. Thin shells made out of reinforced concrete are similar to any other type of concrete construction, except that they are considerably thinner than traditional concrete elements and thus achieve an economy of materials. As opposed to more common concrete forms which can be used together to generate an entire building, thin shells are only used for roofs with large spans relative to their thickness and for walls. Concrete shells are not considered one of the main structural applications for concrete; a typical concrete design textbook will include instructions on how to calculate beams, slabs, columns, footings, and retaining walls, but will not include a section on shells (Nilson *et. al.*, 2010; Meyer, 1996). This leads to a considerable lack of experience in analyzing shells among current engineers, even though they have been successfully built in many different forms.

According to Billington's *Thin Shell Concrete Structures* (1982), there are three distinct levels of structural analysis used in thin shell design. The first type is a simplified analysis in which the behavior of the shell is related to an elementary mechanical form, such as a ring, cantilever, beam, or arch (Billington, 1982). Most thin shells can be conceptualized as either one or a combination of elementary forms, although a designer must justify the validity of an equivalent physical structure through modeling, computer analysis, or by measured observations of full-scale behavior (Billington, 1982). For many pioneering shell designers, a simplified analysis is the only level the designer completed before building structures and learning from this experience. In the eyes of Billington, "the advantage of a simplified analysis is that the designer can spend more time thinking about assumptions, forms, and construction all at the crucial stage of preliminary design" (Billington 1982, p. 3). Simplified analysis emphasizes equilibrium only, but can also consider ultimate loads, buckling, vibration, settlements, creep, and shrinkage if desired (Billington, 1982). The second level of scientific analysis is more rigorous and aims to calculate stresses and displacements in a structure, using these values to maximize efficiency in the structure by sizing elements according to what forces they are able to take (Billington, 1982). From the second level of analysis, it is possible to fully establish dimensions of a structure and complete its reinforcement plan. The third level of analysis, usually restricted to research programs, fills in the gaps left by the first two levels by looking into the more complex behavior of a shell. Today it is usually done by computers, although a third-level analysis is rarely completed on a commonly shaped structure as part of the normal design process. Each of these analyses relies heavily on assumptions about geometry, material properties, and boundary conditions, forcing the engineer to use his or her own judgment and intuition (Billington, 1982).

3.3. Membrane Theory

Generalized numerical theories for shells quickly lead to high order partial differential equations, which is unnecessarily complex for many engineering applications (Draper, 2008). The membrane theory, which was used to predict the behavior of shells before computers were widely used, can simplify analysis into a more manageable process. Designers must be wary of the membrane theory since it does not accurately predict the behavior of all hypar forms, but for many structures it can give a good approximation. The main assumption of the membrane theory is that the shell carries loads through in-plane stresses and does not incur any bending moments or out-of-plane shear forces (Billington, 1982). This assumption leads to three consequences: the shell is statically determinate and can be solved using equations of equilibrium, the boundary conditions of a system must provide edge forces, and the boundary conditions must permit edge translations and rotations which are calculated from equations of equilibrium (Billington, 1982). In shell design that relies only on simplified analysis, an iterative process should be undertaken to find a form in which moments are as small as possible. Iteration to minimize bending results in continuous stress across the section of a shell, allowing the membrane theory to accurately predict the safe behavior of a structure.

In simplified thin shell analysis, the dimensions of a shell are such that it can be defined by the geometry of its middle surface, with internal forces defined as stress resultants and stress couples along this surface (Billington, 1982). On curved surfaces, if a trapezoidal differential element is considered, the equations for the resultants and couples respectively are as follows, where *h* is the thickness of the shell, *r* defines the curvature of the element, *dy* is the depth of the element, *dz* is the thickness of the element, $d\theta$ is the differential angle, and σ represents stress (See Figure 3.1):

$$N_\theta \, dy = dy \int_{-h/2}^{h/2} \sigma_\theta \, dz \qquad (Equation\ 1)$$

$$M_y r \, d\theta = r \, d\theta \int_{-h/2}^{h/2} \sigma_y z \, (1 - \frac{z}{r}) \, dz \qquad (Equation\ 2)$$

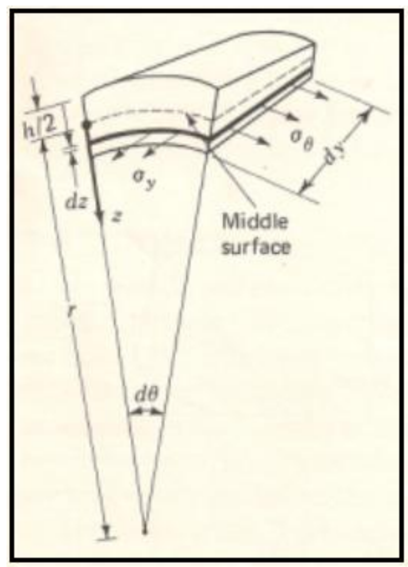


Figure 3.1: Thin shell differential element. Diagram and equations from *Thin Shell Concrete Structures.*

In membrane theory, all $M = 0$, eliminating the need for the second equation and allowing a new variable $N_\theta{}'$ to represent the membrane stress resultant that is in tension when positive and compression when negative (Billington, 1982). In order for these generalized equations to be applied to a specific shell structure, the actual shape must be established, since the derivation is different for each form. For this reason, a further description of membrane theory behavior will be given in the following section on hypars, the central form studied in this thesis.

3.4. The Hyperbolic Paraboloid

3.4.1. Geometry

The hyperbolic paraboloid, or hypar for short, is a doubly curved surface with negative Gaussian curvature (Billington, 1982). It is formed by the sweep of one parabola over another parabola given that the parabolas are perpendicular to one another and open in opposite directions (Draper, 2006). In mathematical terms, a parabola is the set of all points on a two-dimensional Cartesian coordinate plane that are equidistant from a fixed line (called the directrix) and a fixed point (called the focus) that does not fall on the directix. Halfway between the directrix and the focus is the vertex of the parabola, while a line passing between each of these points forms the axis of the parabola (Larson *et. al.*, 1994). The standard equation for a parabola takes the form:

$$(x-h)^2 = 4p(y-k) \qquad\qquad (Equation\ 3)$$

In the above equation, the vertex of the parabola is located at *(h,k)*, the focus is a distance *p* from the vertex along the axis, and the parabola opens along the vertical axis upward or downward depending on the sign of *p*. When two parabolas are combined in three-dimensional space to form a hypar, the equation goes to:

$$z = \frac{y^2}{h_2} - \frac{x^2}{h_1} \qquad\qquad (Equation\ 4)$$

Where

$$\frac{\partial^2 z}{\partial x^2} = -\frac{2}{h_1} \qquad \frac{\partial^2 z}{\partial y^2} = -\frac{2}{h_2} \qquad \frac{\partial^2 z}{\partial x \partial y} = 0$$

In this equation, both h_1 and h_2 must both be of the same sign, or else the equation describes an elliptical paraboloid with positive Gaussian curvature. The variables h_1 and h_2 represent twice the principle radii of curvature of the hypar at the origin (Billington, 1982).

An important aspect of hypar geometry for construction with straight wooden formwork is that the hypar can be formed by two straight line generators. These two generators, which will here be called α_1 and α_2, can be derived by substituting two new variables into Equation 4, the standard equation for a hyperbolic paraboloid (Beles *et. al*., 1976). In order to determine the straight line generators, let:

$$b_1 = -\frac{h_1}{2} \qquad\qquad b_2 = -\frac{h_2}{2}$$

With this substitution, the equation for a hypar can be rewritten as:

$$\left(\frac{x}{\sqrt{b_1}} - \frac{y}{\sqrt{b_2}}\right)\left(\frac{x}{\sqrt{b_1}} + \frac{y}{\sqrt{b_2}}\right) = 2z \qquad (\textit{Equation } 5)$$

This equation yields two systems of straight line generators in three-dimensional space:

$$\text{System 1:} \begin{cases} \frac{x}{\sqrt{b_1}} - \frac{y}{\sqrt{b_2}} = \frac{z}{\alpha_1} \\ \frac{x}{\sqrt{b_1}} + \frac{y}{\sqrt{b_2}} = 2\alpha_1 \end{cases}$$

$$\text{System 2:} \begin{cases} \frac{x}{\sqrt{b_1}} + \frac{y}{\sqrt{b_2}} = \frac{z}{\alpha_2} \\ \frac{x}{\sqrt{b_1}} - \frac{y}{\sqrt{b_2}} = 2\alpha_2 \end{cases}$$

These generators have a number of key properties (Beles *et. al*., 1976):

1. A since generator passes through each point on the hypar's surface.
2. Two generators of the same system never meet.
3. Two generators of two different systems always intersect according to a parametric expression at a point on the hypar surface where:

$$x = (\alpha_1 + \alpha_2)\sqrt{b_1}$$

$$y = (\alpha_1 - \alpha_2)\sqrt{b_2}$$

$$z = 2\alpha_1\alpha_2)\sqrt{b_2}$$

4. The generators α_1 and α_2 are the only ones that occur on the surface.

5. Generators passing through a point on the surface determine the tangent plane at that point.

6. The generators in each system are parallel to the fixed planes:

$$\frac{x}{\sqrt{b_1}} - \frac{y}{\sqrt{b_2}} = 0 \qquad \text{and} \qquad \frac{x}{\sqrt{b_1}} + \frac{y}{\sqrt{b_2}} = 0$$

These straight line generators are useful tools for shell builders in multiple phases of design and construction. First, they make the surface easier to model, visualize, and analyze during the preliminary design phase (Billington, 1982). Second, they can be used to generate formwork for the shell, reducing cost by using straight boards following these equations to create a curved structure (Billington, 1982).

Despite always being defined by this basic geometry, a hypar's structural behavior can vary depending on whether it is bound by curved parabolas or straight edges. For this reason, it is important to delineate the boundaries of a given surface. There are two main ways to define the boundaries, the first of which is to project the hypar onto a given planar shape. The most basic hypar form of the saddle fits into this category, since it is a hypar projected onto a rectangular boundary (See Figure 3.2) (Beles *et. al.*, 1976). In this instance, the corners of the hypar are all

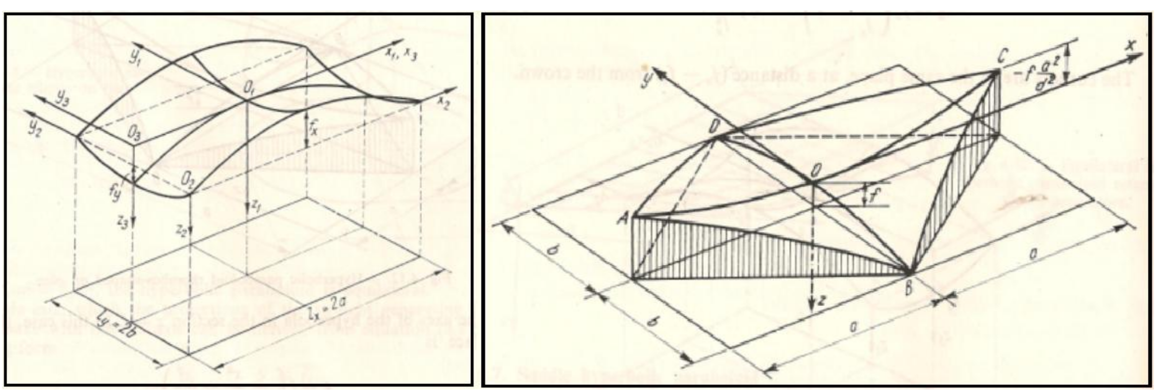


Figure 3.2: A hypar projected onto a rectangle (left) and a rhombus (right). Diagrams from *Elliptic and Hyperbolic Paraboloidal Shells used in Constructions.*

co-planar, and the edges are all curved parabolas along a vertical plane but are straight when viewing the saddle from above. To obtain a soaring edge on a hypar, it is necessary to project part or all of the hypar onto an elliptical boundary, which allows the edge to be curved in all three dimensions (Beles *et. al.*, 1976). Another similar form is generated when a hypar is projected onto a rhomboidal boundary, creating what is commonly called a double cantilever. In this instance, only two of the corners are on the same plane, while the other two are translated the same distance away from the projection plane (See Figure 3.2).

There are also a number of ways in which hypars can be cut and combined to form a finite surface, which does not have to be continuous once completed. A third basic hypar form, called a tympan, is the basis of many of these combinations and it consists of an equilateral hypar referred to its generators (See Figure 3.3). This shape can be visualized as a warped

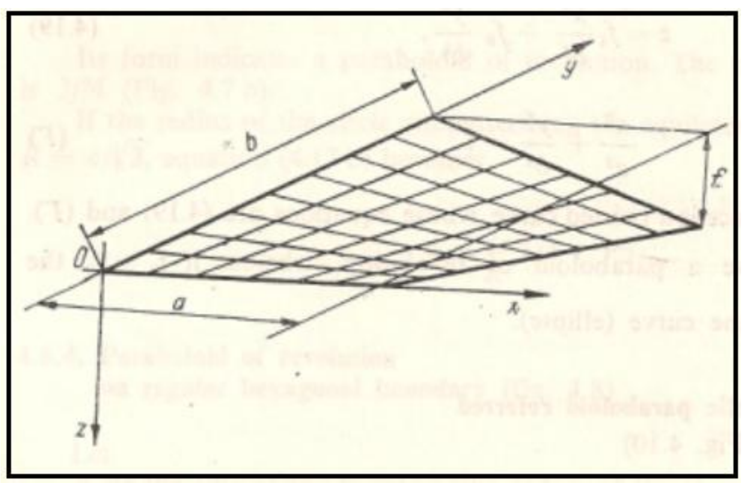


Figure 3.3: An equilateral hypar referred to its generators. Diagram from *Elliptic and Hyperbolic Paraboloidal Shells used in Constructions.*

plane created out of four rectangular corner points in which one of the corners has been translated perpendicular to the plane, leaving completely straight edges on all sides. The two outside edges remain in the same plane, but the other two edges slope down to meet the non-coplanar point. A tympan is often combined with other tympans at its edges to create the visual effect of inverting the hypar, which will be described in detail later. A radial tessellation of four identical equilateral tympans around their non-coplanar points forms a basic umbrella shape consisting of a square projection and four folds that occur between sections. Intersecting paraboloids, often called gabled or groin vault roofs when referring to shells, would also fit into this category of combined hypar pieces. A gabled roof is made of four tympans combined at their coplanar points, and a groin vault is made of combined saddles. Each of these hypar configurations has been built in reinforced concrete, and examples of each are discussed in the next chapter.

3.4.2.Membrane Behavior in Hypars

In order to find the true stress resultants imposed axially on doubly-curved shells, Billington found it is more convenient to study the membrane values projected onto a horizontal *xy*-plane below the shell rather than derived them directly from the curved differential element described earlier (Billington, 1982). When working with this projection, Billington applies a generalized membrane theory for translation shells of double curvature to specific hypar structural forms, obtaining normal and shear stresses in different regions of the shell. These derivations are in some instances long and complicated, and they are numerous owning to the many possible configurations of hypar shells. As such, an exhaustive listing of all derivations is not useful for the purposes of this thesis. However, in order to demonstrate how stresses are calculated according to membrane theory, an example on one configuration will be given.

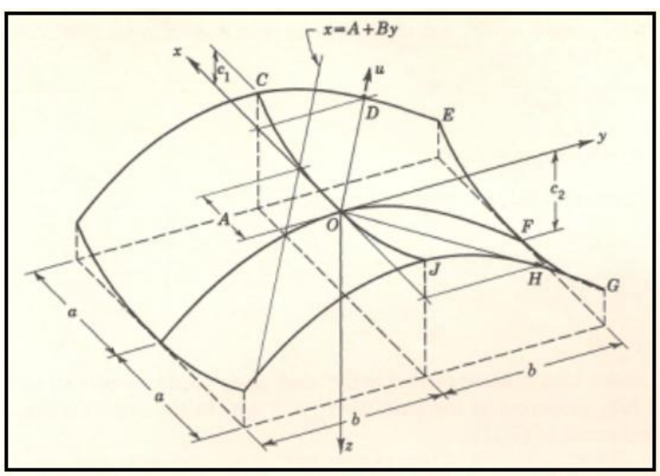


Figure 3.4: The hypar saddle geometry used for this derivation. Diagram from *Thin Shell Concrete Structures.*

Consider the instance of the basic hypar saddle bound by its projection onto a rectangular surface (See Figure 3.4). Billington's general differential equation governing the stress function of a doubly-curved shell is as follows, where q is the distributed load and F is the stress function (Billington, 1982):

$$\frac{\partial^2 F}{\partial x^2}\frac{\partial^2 z}{\partial y^2} - 2\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 F}{\partial y^2}\frac{\partial^2 z}{\partial x^2} = q \qquad (Equation\ 6)$$

When the standard equation for a hyperbolic paraboloid (Equation 4) saddle is substituted into this equation, the expression becomes:

$$\frac{\partial^2 F}{\partial x^2}\frac{2}{h_2} - \frac{\partial^2 F}{\partial y^2}\frac{2}{h_1} = q \qquad (Equation\ 7)$$

If the case of a simple uniform loading over the horizontal projection (i.e. gravity with a constant weight of the structure) is taken, then $q = -p_z$, and there are three possible stress functions F which can be considered to solve the differential equation (Billington, 1982). These functions are:

$$F = -\frac{1}{4}p_z h_2 x^2$$

$$F = +\frac{1}{4}p_z h_1 y^2$$

$$F = -\frac{1}{8}p_z (h_2 x^2 - h_1 y^2)$$

The three different stress functions yield the following three results, respectively:

$$N_x = 0;\ N_y = -\frac{p_z h_2}{2}; N_{xy} = 0$$

Where the entire vertically distributed load is being carried in the shell along parabolas parallel to the *yz*-plane;

$$N_x = +\frac{p_z h_1}{2};\ N_y = 0; N_{xy} = 0$$

Where the entire vertically distributed load is being carried in the shell along parabolas parallel to the *xz*-plane; and

$$N_x = +\frac{p_z h_1}{4};\ N_y = -\frac{p_z h_2}{4}; N_{xy} = 0$$

Where the forces are equally split between the two systems of perpendicular parabolas. These three different solutions may seem arbitrary, but the choice of which one to use depends on the edge supports, highlighting the importance of a structural designer in choosing the right form (Billington, 1982). If there is full arch support on the edges parallel to the *y*-axis but not the *x*-axis, the first solution is accurate. If this condition is reversed, then the second stress function provides the most accurate solution. In the case where both edges have the same stiffness (such as with free edges), then the third solution is what should be used, showing that the loads are shared equally by each set of parabolas (Billington, 1982).

A derivation for hypars with straight edges such as an umbrella will result in a similar system of solutions, although the edges are assumed to be of the same stiffness, eliminating two stress functions (Billington, 1982). In this case, membrane theory also shows that a vertically distributed load is carried by pure arch action through the shell. Effectively, in a simplified version of the shell there are two systems of arches present: tension arches running between the two coplanar edges, and compression arches perpendicular to the tension ones, running from the coplanar edges down to the umbrella folds (See Figure 3.5) (Billington, 1982). In any case, both of these derivations illustrate the difficulty in directly calculating the structural behavior of thin shells. Even while using the membrane theory, the derivations involve high order partial differential equations. For this reason, many past shell designers searched for simple analogies that could accurately predict the behavior of shells to a reasonable degree, verifying their analyses with computer models or full scale structures. Two of the most useful of these analogies are given in the next two sections.

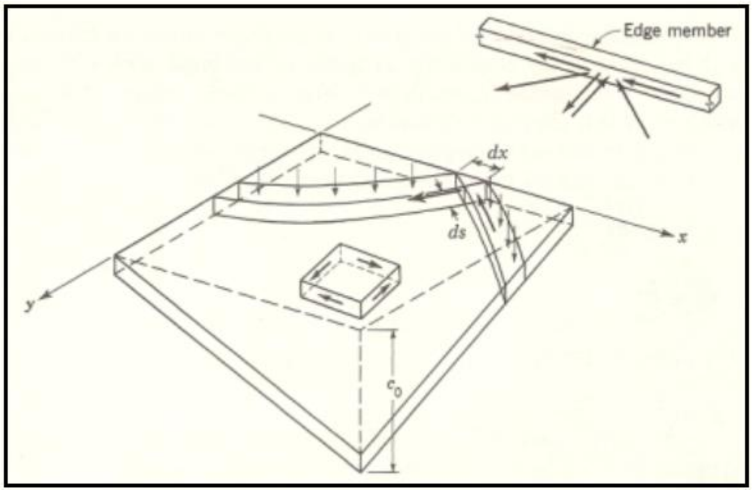


Figure 3.5: Visualization of tension and compression arches in hypars with straight edges. Diagram from *Thin Shell Concrete Structures.*

3.4.3.Cantilever Analogy

In terms of structural behavior the hypar cannot be thought of as only one form, and as of the 1980s Billington had defined at least four well-developed types of hypar behavior (Billington, 1982). Two of these forms, the double cantilever and the inverted umbrella, essentially behave as a cantilevered beam. Since pieces of these structural elements are used by a number of different hypar configurations at full scale, the behavior of many hypar shells can be predicted by using a cantilever analogy. The cantilever analogy fits into Billington's first level of analysis, and can thus be a powerful tool in the preliminary design phase (Billington, 1982).

According to the analogy, an umbrella shell (in the form of one or multiple tympans) acts primarily as a triangularly shaped cantilever branching out from the central support (Pazdon, 2009). A uniformly distributed load is carried to the central support as a couple, with the free edge of the umbrella being in tension and the fold, or valley, being in compression. The stresses in the middle of the shell are so small that they are inconsequential compared to the stresses at the folds and along the edges. The magnitudes of both the tension and compression resultants can be estimated directly from the dimensions of the umbrella and the load (Pazdon,

2009) (See Figure 3.6). The cantilever analogy highlights an important structural reality when designing umbrella shells: in hypar surfaces with straight edges, the tension and compression

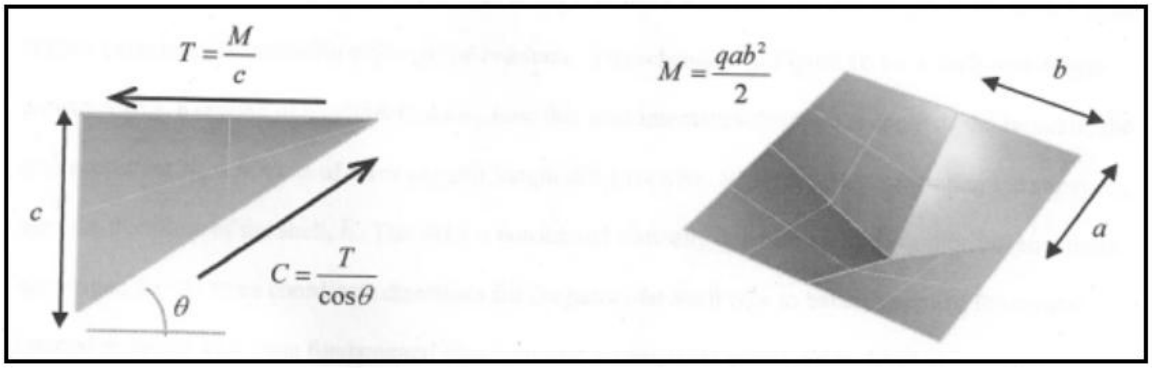


Figure 3.6: Diagram of the cantilever analogy, courtesy of Jennifer Pazdon.

arches of a shell carry all loads to the edges, where they are resolved into edge shears, requiring edge members that are loaded axially (Billington, 1982). The axial behavior of edge members occurs when two hypars are butted against one another as in an umbrella, causing the components of the forces perpendicular to these edge members cancel each other out, and the components parallel to the edges add to one another, putting stress only in the direction of the edge member (Billington, 1982). An understanding of this behavior can govern the preliminary shape design of hypar thin shells in a number of different configurations.

3.4.4.Stiffened Arch Analogy

For hypar forms in the both the saddle and gabled roof shape, the cantilever analogy is of no use, since there is no central column to carry all of the loads. Instead, these hypar configurations can be thought of as a series of stiffened arches which carry a vertically distributed load through purely axial forces with a constant horizontal component and a varying vertical component across the shell (Billington, 1982). To understand this analogy, consider a simple, square groin vault made of intersecting hypars (See Figure 3.7). In both parabolic cylinders of this shell, arch strips carry verticals loads axially down to the groin, at which point they join forces with similar but perpendicular arch strips from the other cylinder. This has the cumulative effect of loading the diagonal arch with axial forces, since opposite direction components cancel out and same direction components add (Billington, 1982).

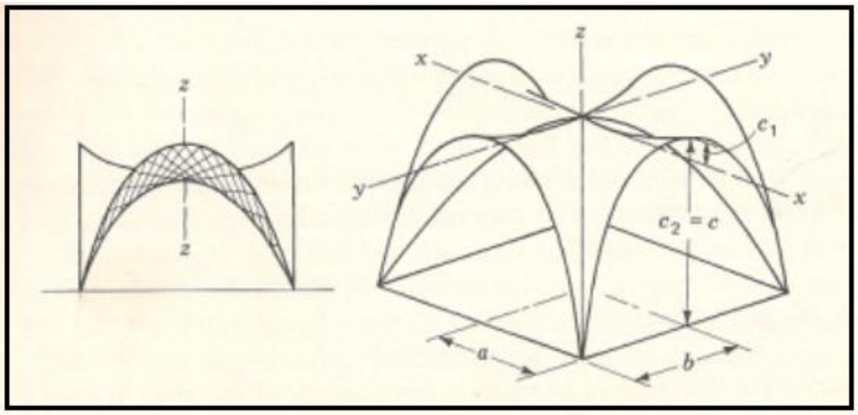


Figure 3.7: A simple hypar groin vault. Diagram from *Thin Shell Concrete Structures.*

These diagonal arches then carry all of the horizontal and vertical components of the distributed load down to the corner supports as if they were three-hinged arches. Hinge behavior is possible because at the crown of the structure, the groin loses all of its depth and is only able to resist bending with the depth of the shell, effectively turning it into a hinge. The supports are often hinged as well to avoid bending forces in the bottom of the shell (Billington 1982). Using static equilibrium, it is possible to show that no bending occurs in the groin arch when it is assumed to be three-hinged, allowing for simple calculation of stresses and reactions. Finite element analyses of groin vaults have shown this simple analogy to be reasonably accurate in predicting shell behavior (Pazdon, 2009). Other hypar forms that use the arch analogy, such as the gabled roof, behave slightly differently than the groin vault example given here, but the method of using arch calculations in the preliminary design phase is similar. The stiffened arch analogy is limited in that it does not take into consideration deformations or questions of creep, cracking, and support movement, but an equilibrium analysis using this analogy can be useful for engineers in the early form-finding stages of a design.

3.5. Folded Plates

Folded plates are a method of designing concrete shells that takes advantage of the stiffness provided by shells meeting at acute angles to one another, analogous to folding a piece of paper. Although folded plates are distinct from hyperbolic paraboloids in that they use straight rather than curved elements, a brief description of their structural behavior is given to provide background information for the Miami Marine Stadium. In the Marine Stadium, the engineer borrowed elements typical of folded plate design and added them to hypar geometry, creating a hybrid form between the two systems. The two types of concrete design are able to work well together because the force flow exhibited in folded plates is similar to that of hypar umbrellas. In a folded plate roof, loads are taken down to columns or walls primarily by stiffening elements such as edge beams and thickened folds between plates (Ketchum, 1997).

A folded plate concrete roof contains a number of basic elements: inclined plates, edge plates or beams, stiffeners, and supports (Ketchum, 1997). The inclined plates are the thinnest part of the shell, responsible only for covering a space and transferring their weight to a fold or edge member. Edge plates and beams are then responsible for taking the majority of the loads, either in tension or compression depending on placement. For an example of how this works, consider a folded plate 'V' structure acting as a simply supported beam in its longitudinal

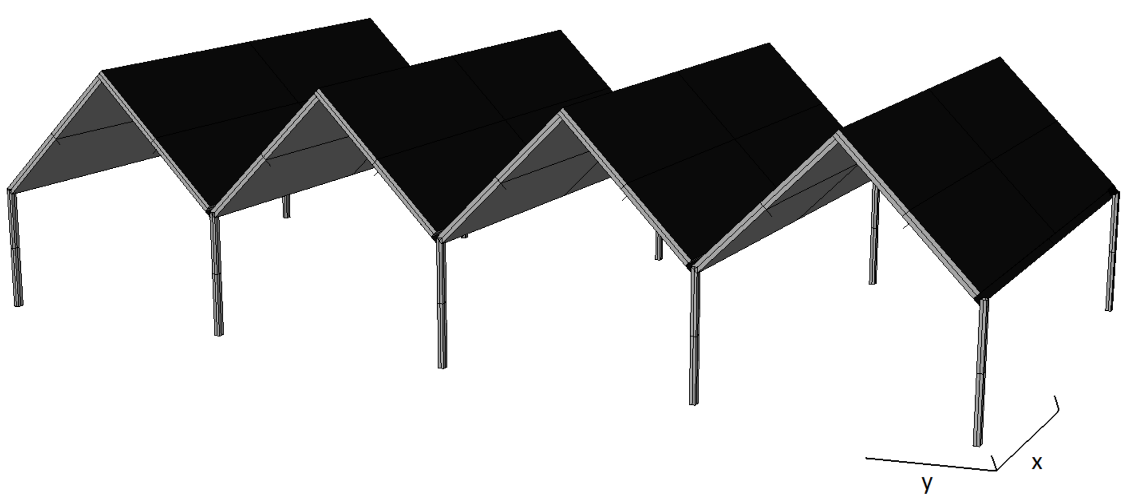


Figure 3.8: A simply supported folded plate roof.

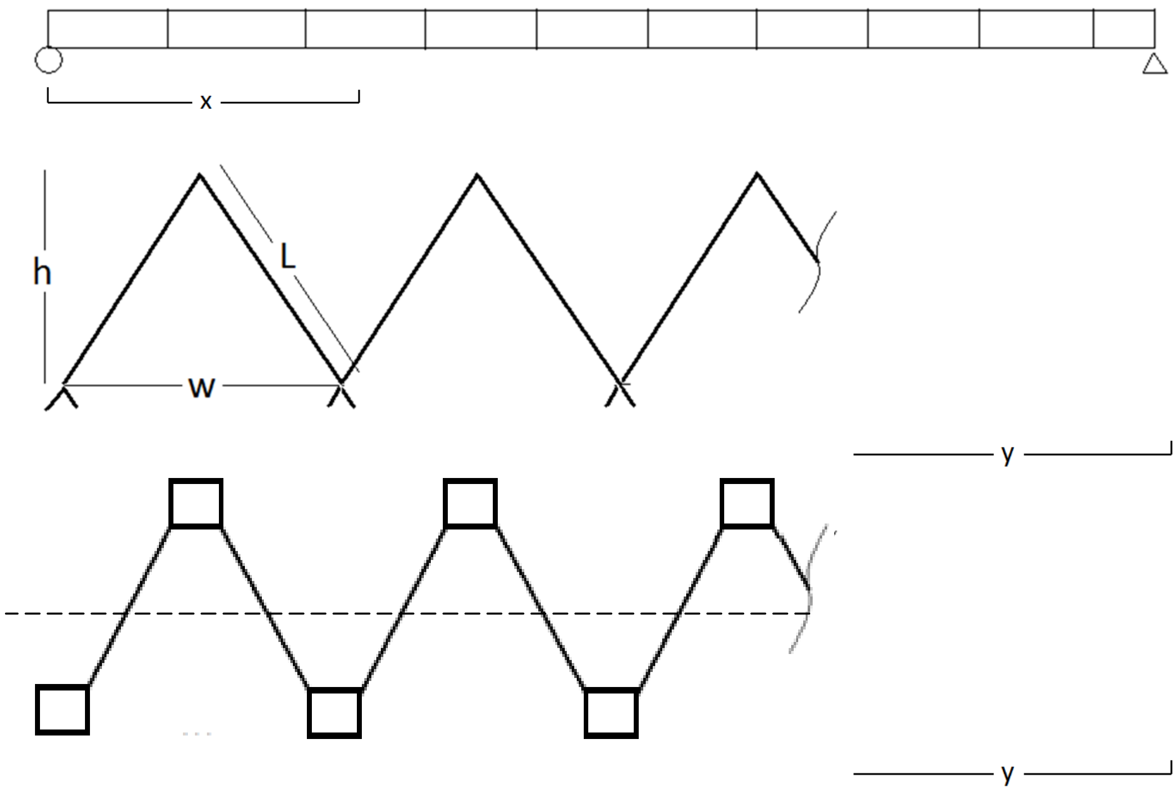


Figure 3.8 (continued): A folded plate roof, with approximations of structural behavior.

direction (see Figure 3.8). In this setup, the shell is designed to be thicker at the folds, leading to the approximation given below the original geometry. According to equilibrium equations, when the beam is given a continuously distributed load along its length in the *x* direction, a portion of the shell above its *y* direction neutral axis will be in compression, and the bottom of the beam will be in tension. A simplifying assumption can be made to ignore the contribution of the shell and assume the entire top folds take the compression resultant and the entire bottom folds take the tension resultant. A similar method of calculating folded plate beams is used later in verifying finite element models of the Miami Marine Stadium. In addition to the inclined planes and edge members, a folded plate structure will generally included stiffeners to hold the folds in place and prevent large deflections (See Figure 3.9) (Ketchum, 1997).

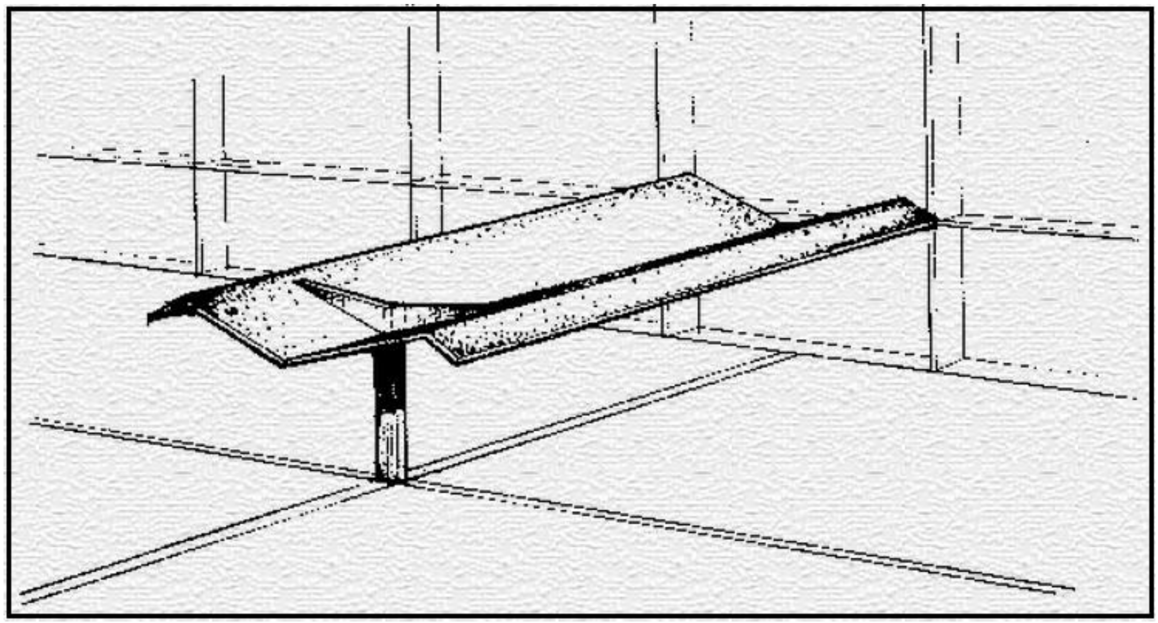


Figure 3.9: A stiffener element on a folded plate roof. Image courtesy of Mark Ketchum.

The main advantages of a folded plate system is that plane surfaces are easier to create with formwork than many curved shapes, and yet still achieve an economy of materials compared to bulkier concrete forms (Ketchum, 1997). However, the increased ease of construction provided by planar forms comes at a price, in that plates are often not as structurally efficient as arched forms for roofs. The structural analysis of the Miami Marine Stadium in Chapter 6 acts as a case study for direct comparison between the two methods, illustrating the superior efficiency of curved forms. Furthermore, folded plates are also not capable of very large spans on their own when compared to curved shapes such as a hypar umbrella or barrel vault (Ketchum, 1997). Despite these limitations, folded plates are still a reasonably efficient and cost effective choice for many structural applications.

3.6. Construction Technique for Hypar Shells

A variety of construction techniques for hypars, both old and new, are discussed throughout different sections of this thesis. These range in sophistication from nailing boards together and pouring concrete on top to projecting wet concrete at high velocity towards inflatable membranes. This section does not cover all possible methods, because it is not meant to be exhaustive. Rather, its purpose is to give background information on how a hypar shell is usually built in basic terms. Particular emphasis is given to the simple methods of construction used historically by thin shell designers to build cheap, efficient structures.

The first step in the construction of a concrete shell is to build scaffolding, sometimes called falsework, which can be either wood or metal. The axial members of falsework support the actual form boards that give shape to the shell, which are built next and placed on top of the falsework. These form boards can be either flat or curved; however, if they are flat pieces of plywood, the cost is reduced substantially (Garlock *et. al.,* 2008). In formwork for a hypar, relatively small and thin boards do not require substantial bending and twisting, allowing for the possibility of straight form boards to create a curved structure (Garlock *et. al*., 2008). The dimensions of the falsework and formwork can be calculated using the straight line generators that define the geometry of the shell. In addition, the pieces of scaffolding can be attached and supported by any means that are effective, since scaffolding and formwork simply need to hold the loads of the concrete until it hardens, after which point it will be discarded or recycled.

Once the formwork is in place, steel reinforcing is laid out over the future position of the shell. Since shells are so thin, it is common to build a grid pattern of small steel rods over large areas of the shell rather than to have reinforcing in both the top and bottom. The single steel layer is not the case for the edges, supports, and connections, which have more concrete available for cover and in which the designer must place considerable amounts of steel to help the concrete take large loads. After the steel has been tied together and is in position, the shell is ready for the

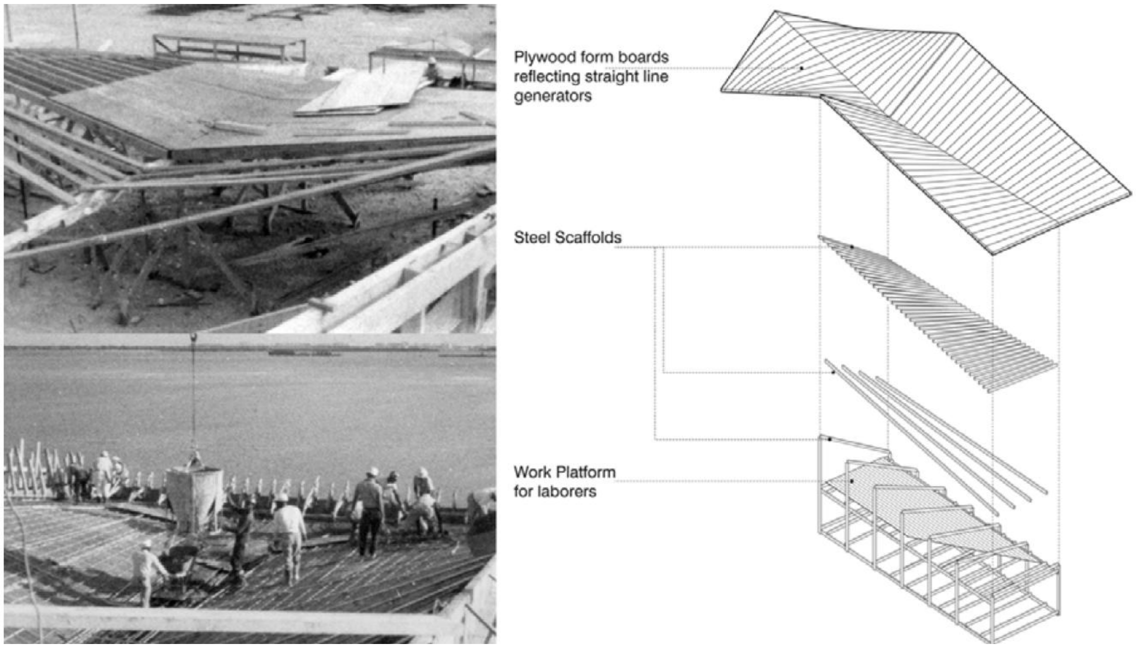


Figure 3.10: An example of hypar shell construction given by the Miami Marine Stadium. Images courtesy of Sigrid Adriaenssens.

addition of concrete. At this point, the designer may make additional changes to further prepare the structure, although many additions are simply preferred rather than necessary. For example, Félix Candela would often pour a small layer of cement grout over the form to make sure that the final structure had a smooth interior finish, since the wet concrete of thin shells is typically not vibrated (Garlock *et. al*., 2008).

After preparing the reinforcing, concrete is poured directly in place and left to harden. The pouring process can be done effectively in a number of different ways, with the most simple being the use of wheelbarrows, buckets, and manual spreading tools on top of the shell. This is the method preferred by older shell builders like Candela, and also what was used on the Miami Marine Stadium. Once the concrete hardens, the formwork must be removed in a delicate process called ‘decentering’. Decentering must be done carefully and evenly, ensuring a balanced removal that will not cause localized stress in thin regions of the shell not designed to take them. In many thin shell designs, the falsework and form boards are taken down and then reused on another identical section of the shell. Once the concrete hardens and all other material is removed, the shell is essentially finished, able to stand on its own. A shell may deflect a large amount during the removal of form boards and may continue to deflect at a smaller rate due to time-dependant aspects of structural behavior. Once subjected to loads and weather, a shell is also susceptible to cracking, spalling, delaminating, or other small types of deterioration. However, as long as these deflections and deterioration are reasonable, a shell will require minimal maintenance over the course of its lifetime (Draper, 2008).

3.7. Conclusion

A technical understanding of shells was developed gradually throughout the twentieth century, with contributions made by both theoretical propositions and full scale experiments. Although early developments in shell theory were restricted to simple geometric shapes such as circular arches and domes, a few engineers quickly realized the mathematical background necessary to understand these shape exceeded the comfort zone of many practicing engineers. Thus, they sought out theoretical simplifications, accurate analogies, and specific shell forms such as the hypar that were well-suited to simplified analysis. In many cases, membrane theory or the cantilever analogy was all an engineer needed to justify his structure, with the final proof coming in the form of a strong, full-scale shell. Most engineers, however, were not granted the freedom to experiment as aggressively as some of the thin shell masters. For this reason, a technical understanding of the structural behavior of shells is still necessary.

4. Classification and Performance Evaluation of the Hypar Form

4.1. Introduction

As expressed by Professor David Billington in his introduction to *Thin Shell Concrete Structures*, there is value in understanding and assessing the structural engineering solutions that have been offered in the past. As opposed to science, which is fundamentally concerned with a systematic, thorough understanding of how the universe works, structural engineers must first choose a form, and then make sure it works (Billington 1982, p. 1). If it does not, the options are either to perform an iterative design process, or to start again with a new idea. Contemporary advances in structural formfinding notwithstanding, there are important lessons in the idea Billington conveys. An understanding of what has worked in the past can save time and heartache on the front end of the structural design process. For this reason, a historical approach to engineering can be valuable and lead to better designs in the future. A solid foundation in the history of structures does not need to be restrictive or constraining, since new ideas and innovations are always possible. Similarly, a historical understanding of structures should not lead to a dogmatic defense of a certain style, as has often happened in the architectural world. Being cognizant and cautious of these two possibilities, this chapter does not aim to posit hypar shells as the only way to build in concrete correctly, nor is it a polemical defense of the thin shell style. Rather, it is hoped that this history serves as a starting point in the mind of the designer, who can choose to imitate and improve upon a given form or to think about a structural problem in an entirely new way, starting his or her own stand of engineering history.

In order to get the most historical value out of existing structures, this chapter proposes a classification system designed to highlight key relationships between the many examples of hypar shells scattered across the globe. The list of examples is not meant to be exhaustive, but rather to serve as a field of reference for discussing the properties of different types of shells. The system classifies structures based on configuration, which has ramifications for both structural behavior and architectural use. In some instances, examples will be given that are not concrete shells, but still take the hypar form and are comparable enough in scale and function to add value to an understanding of possible shell uses. Each configuration is discussed in terms of its form, use, and economic and environmental efficiency. In addition to these categories, a number of design trends are traced throughout the entire classification system and discussed in terms of their individual importance and relationship to other common trends. The goal of identifying trends is to highlight aspects of hypar shells that can be useful in a number of different spatial or programmatic applications, regardless of a shell's specific configuration.

This chapter also includes a brief evaluation of the sustainability of different hypar configurations. Although many traditional metrics of building sustainability cannot be accurately applied to concrete shells and do not fall under the influence of the structural engineer, there are two clear ways in which a good structural design can improve the sustainability of structures: durability and a reduction of materials. The material usage aspect of shells is analyzed quantitatively by assessing the amount of emissions reductions that can be directly traced to more efficient design. There is a substantial amount of embodied energy in the production of concrete, and thus a reduction of materials in a structure's design can have a positive environmental impact when substituted for more traditional concrete forms. This relationship between structural form and indirect carbon emissions will be explored through a combination of modeling, research, and calculations.

4.2. Classification of the Hypar Form

Figure 4.1 shows the proposed categories for the hypar form, along with a few examples that fit into each category. There are nine major categories identified as distinct applications of the hypar form, although some of these categories can be broken into smaller divisions. Three of these categories are in the cantilevered beam category; four behave primarily according to the stiffened arch analogy; one category is a structural outlier and behaves as a cone wall; and the final category can have elements that behave as either beams, arches, or a combination of both. A majority of the chosen examples are Félix Candela structures built in Mexico, which is expected considering his status as a master of the hypar form. However, a number of the chosen examples are the work of less famous designers and exist either in the United States or Europe. Each of these configurations has unique geometric, structural, and/or programmatic characteristics when employed in a thin shell, and these distinct properties can be clearly scene in the selected examples.

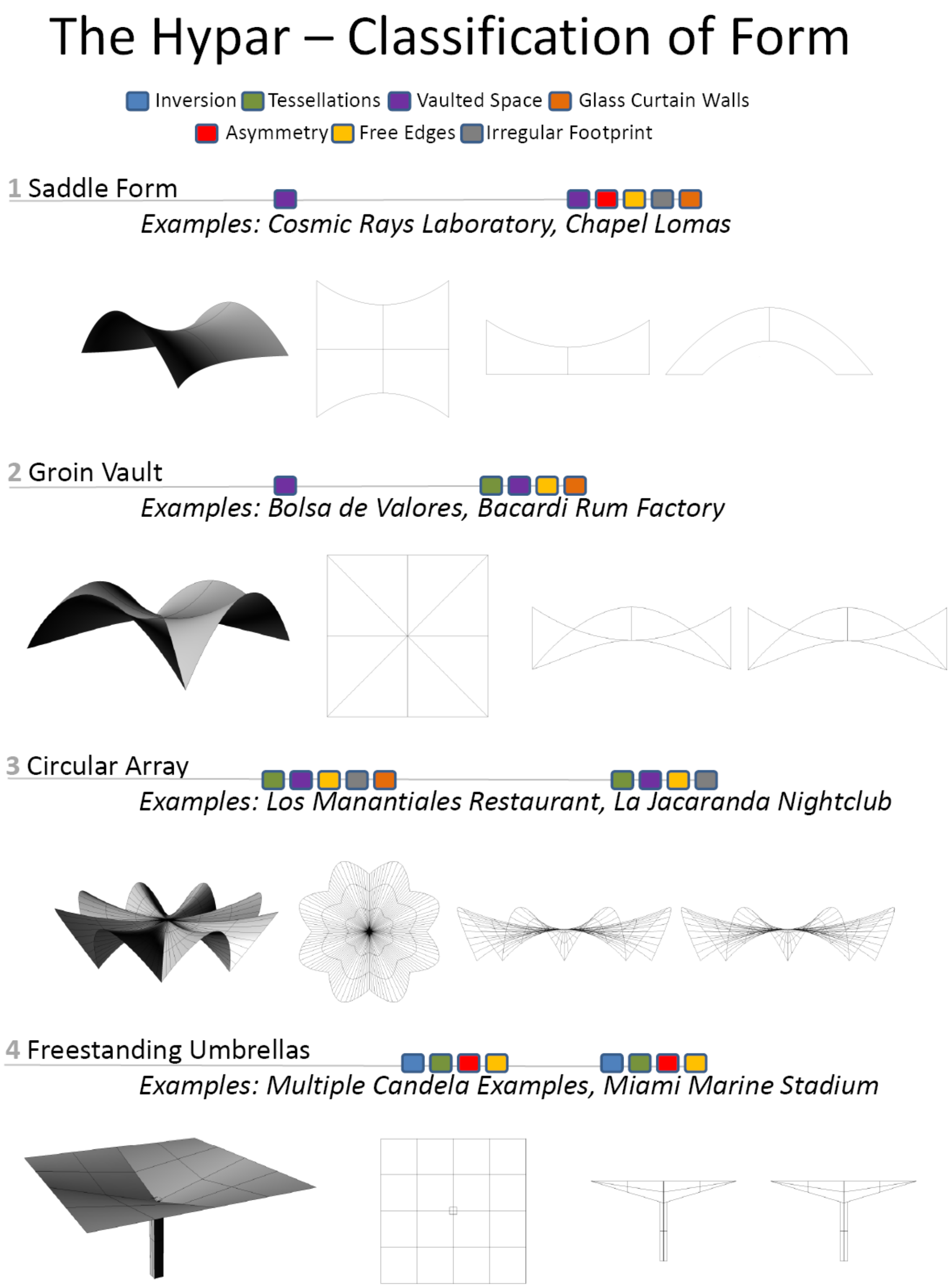


Figure 4.1: Classification of Hypar Form

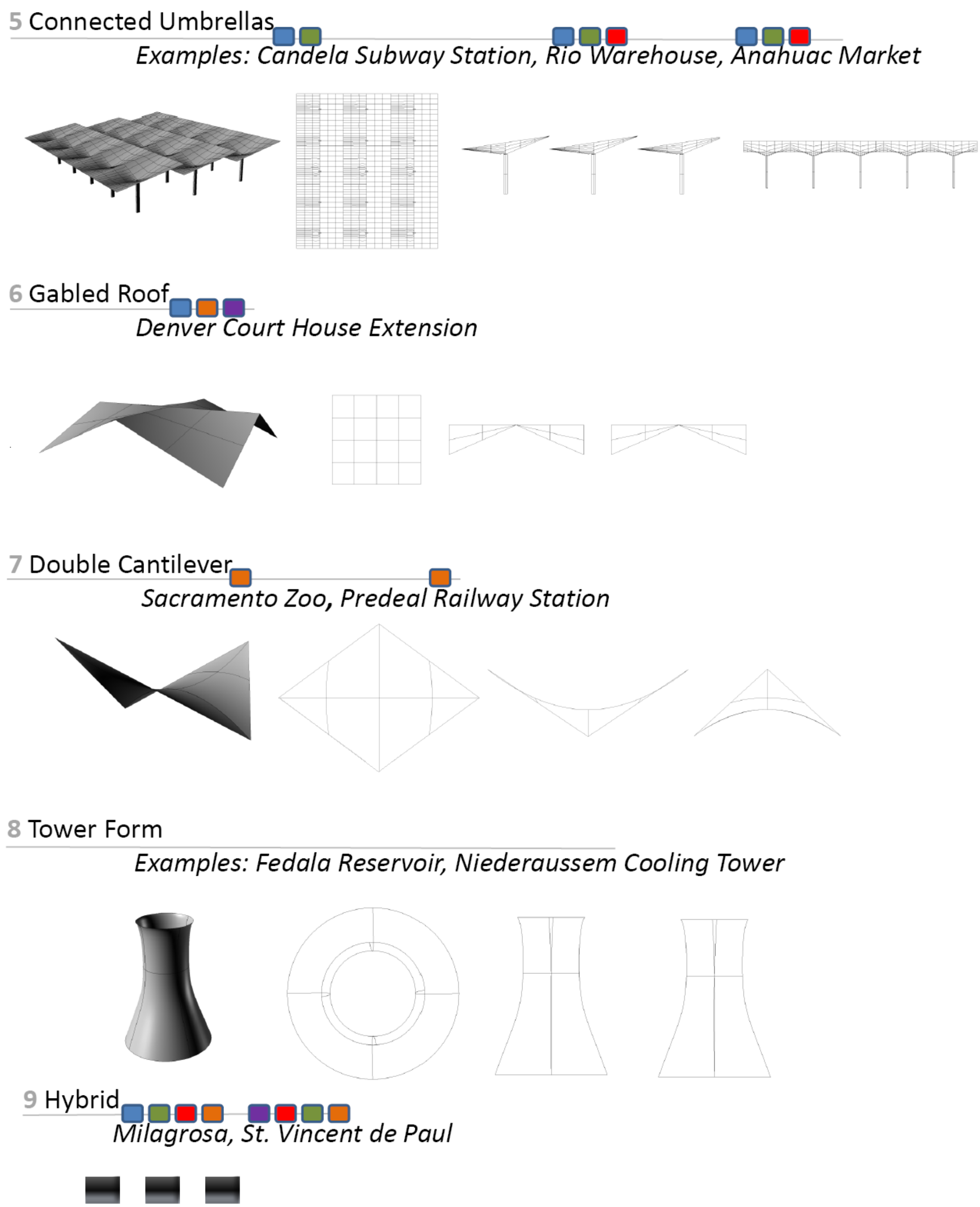


Figure 4.2: Classification of Hypar Form (continued)

The most basic shell manifestation of the hypar form is the saddle shape, which ranges in application from an efficient barrel vault alternative to an asymmetrical saddle with a soaring free edge. One of the earliest examples of the saddle configuration is given by Félix Candela's Cosmic Rays Laboratory, built in 1951 (Garlock *et. al.*, 2008). Tasked with building a laboratory thinly roofed enough to permit the measurement of cosmic radiation, Candela took two back-to-back barrel vault shells and gave them a slight double curvature, allowing for the roof to be as thin as 1.6 cm in some places (Garlock *et. al.* 2008). The result is a distinctive hypar surface, which carries the weight of the shell towards three thick stiffening arches at both ends and the middle of the structure (See Figure 4.3). Although the presence of stiffening ribs shows a hesitation to abandon traditional concrete construction principles due to a lack of experience, the Cosmic Rays Laboratory served as a step towards greater empirical understanding of doubly curved shell behavior, and it paved the way for more innovative structures that came later. Functionally, placing a hypar in this horizontal saddle allows for a traditional vaulted space with a reasonably regular footprint. However, the configuration has the added advantages of reducing material per usable space, eliminating the need for stiffening arches and creating the possibility of thinner roofs when compared to normal concrete barrel vaults.

Figure 4.3: Cosmic Rays Laboratory by Candela. Photo courtesy of Princeton University Art Museum.

A more developed example of the saddle shape is given by a later Candela structure, the Chapel Lomas de Cuernavaca (Garlock *et. al.*, 2008). This Mexican chapel has a more aggressive double curvature, leading to an irregular, curved footprint. It also expresses asymmetry in the saddle, which combines with this steep curvature to result in a soaring free edge (See Figure 4.4).

Figure 4.4: Chapel Lomas de Cuernavaca by Candela. Photo courtesy of the Princeton University Art Museum.

Figure 4.5: Interior of the Chapel, showing the framing of the alter by the shape of the shell. Photo courtesy of Maria Garlock.

The tradeoff for this free edge is that with the steep curvature, there is also considerably less interior space. However, Candela uses the curve to create the visual effect of a small, light opening in the back of the chapel contrasting with the heavily shadowed stage and alter (See Figure 4.5). Chapel Lomas represents a strong innovation within the saddle category, while still retaining the configuration's main properties of vaulted space, no stiffening arches, and a thin roof made possible by the force flow through its double curvature.

The saddle shape is closely related to the double cantilever shape, which is simply a saddle with its edges trimmed and only two main supports. However, the structural behavior of a full saddle, the visual effect of curved edges instead of straight edges, and the shape's property of open-end, horizontal vaulted space make the saddle shape a distinct category. A detailed description of the saddle form's structural behavior is included in the previous chapter, and will not be repeated here. For general reference, both the Cosmic Rays Laboratory and the Chapel Lomas behave as a series of stiffened arches, carrying forces through the shell to the edges, which then carry them down to the ground. A saddle shell can behave differently depending on the placement of its openings, and in both Cosmic Rays and Chapel Lomas only two ends are open, causing it to behave as a doubly curved barrel arch (Billington, 1982).

The second main category of hypar configurations is the groin vault. Although the groin vault is closely related to the circular array in terms of structural behavior, it will be considered its own configuration due to its ability to cover a regular, rectangular plan, allowing for the tessellation of entire vaulted sections. Essentially, a groin vault is the combination of two perpendicular saddles, open or walled off on each of its four sides. The basic, singly curved groin vault form has been around since Roman times and was used extensively in stone by the Gothic and Renaissance architectural traditions. However, adding double curvature allows groin vaults to be made much lighter, reducing the amount of materials needed to support the structure's loads. Many modern architects and engineers noticed the possibilities afforded by thinness and lightness; notably, architectural theorist Sigfried Giedion identified thin shells as a possible solution to the vaulting problem of modern times in his landmark text, *Space, Time and Architecture* (Giedion, 1941). As a tribute to the range of architectural possibilities allowed by hypar shells, Giedion mentioned Félix Candela directly during a foreword to a later edition of his book.

An early example of a hyperbolic paraboloid groin vault is Candela's Bolsa de Valores, a roof built in 1954 for Mexico City's Stock Exchange (See Figure 4.6)   This structure posed a difficult design problem in that it was rectangular instead of square, causing the straight-line

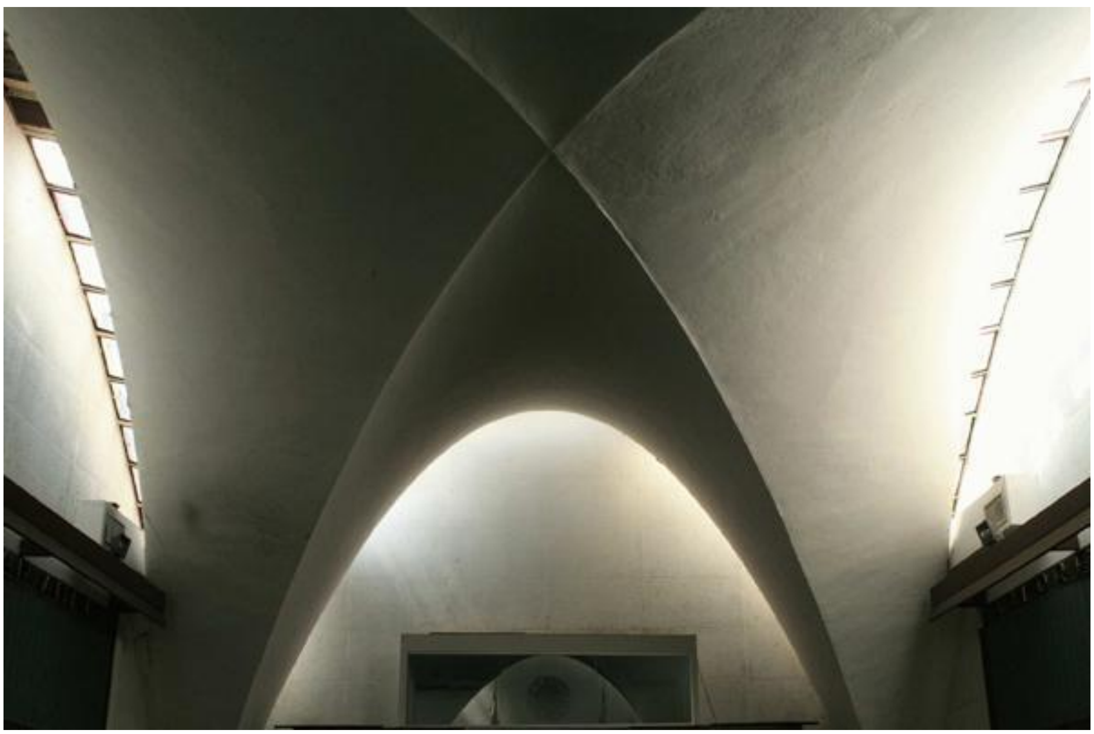


Figure 4.6: Interior of the Bolsa de Valores roof built by Candela. Notice the groins are not perpendicular, which lead to difficulty during construction. Photo courtesy of *Acción Cultural Española*.

generators of the two intersecting hypars to be oblique to one another, further leading to an imbalance of forces in the groins (Garlock *et. al*., 2008). Candela had to include large stiffening edge ribs to take this imbalance, which substantially affect the appearance of the structure. The Bolsa de Valores is also different from many other groin vaults because it stands alone, being formed by only one set of intersecting hypars. Learning from the difficulties and limitations of its design, Candela altered both the oblique and standalone properties of Bolsa de Valores in his church at San Antonio de las Huertas (Clark, 2009). The San Antonio church was built two years later, and its design leaves free edges on the hypars and places three groin vaults in a line (See

Figure 4.7: Photo from the roof of Candela's church at San Antonio de las Huertas. Only two of the three groin vaults are shown, because the photographer is standing on the third. Photo courtesy of Nanette Clark.

Figure 4.7). However, Candela did not leave the edges visually free, instead including a small, thin, arched barrel section offset over the gap where two groin vaults meet. By placing glass in this gap, Candela allowed natural lighting to produce a stunning interior architectural effect (See Figure 4.8). In very different ways, the stock exchange building and the church represent two innovative expressions of a hypar groin vault configuration, indicative of the many ways in which the hypar groin vault can be successfully implemented both structurally and architecturally.

Figure 4.8: Interior of the Chruch at San Antonio de las Huertas, showing the visual effect produced by the staggered shells. Photo courtesy of *Flickr Commons*.

Slightly later in his career, a more experienced Candela designed one of the largest, most famous, and visually distinct hypar groin vaults in existence. This structure, a factory built for the Bacardi Rum Corporation in 1960, was designed as a direct critique of the St. Louis Airport Terminal. The airport terminal was built by Tedesko a few years earlier, and it included large stiffening ribs in both the groins and edges (Garlock *et. al*., 2008). Candela sought to show how stiffening ribs are not necessary, and that double curvature makes a thin shell more structurally efficient. The Bacardi Rum Factory has an imposing size comparable to the shell at St. Louis, with each of the six vaults containing an edge span of 26 meters and a shell thickness of only 4 cm (Garlock *et. al.*, 2008). The factory is light, airy, and visually striking with its skylights and glass curtain walls filling the arched openings and gaps between shells (See Figure 4.9). Due to the size of the factory, Candela was not able to completely support the roof without edge stiffeners, but he was able to hide them from both the inside and ground outside by setting them back from the edge, leaving the edges free to express the structure's thinness (Garlock *et. al.,* 2008). A detailed explanation of a hypar groin vault's structural behavior is included in the previous chapter, but in simple terms, the double curvature of the hypar shell successfully allowed for considerable thinness and the reduction of size of stiffeners when compared to the cylindrical groin vaults of the St. Louis Airport terminal (Garlock *et. al.,* 2008). With this factory, Candela leaves yet another innovative hypar structure, further proving the versatility of the form.

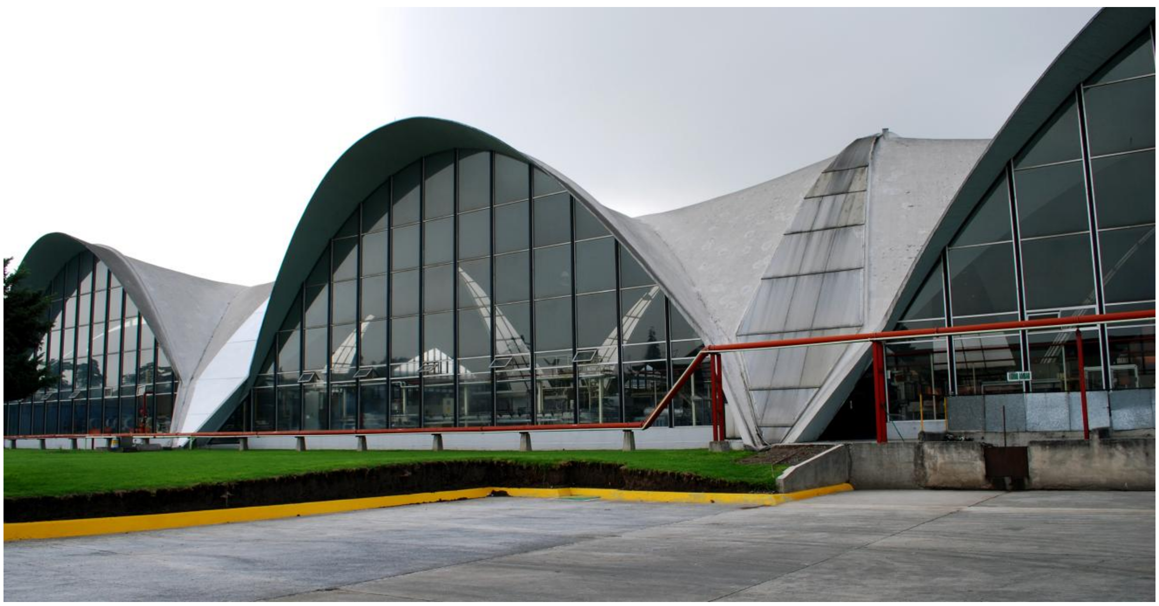


Figure 4.9: Candela's Bacardi Rum Factory shown from the outside, where the free edge is clearly visible. Photo courtesy of The Society of Architectural Historians.

Another major configuration, the circular array, is a groin vault made of intersecting hypars that do not meet at right angles. Again, the distinction is made from the groin vault category because a circular array is used in a programmatically different way, often free-standing and open with an irregular footprint. A circular groin array also gives the advantage of being able to reuse formwork or combine prefabricated pieces, although in some shells it is easier to lay the formwork for the entire structure to achieve balanced forces upon its removal (Garlock *et. al.,* 2008). Structurally, the configuration's behavior is similar to a square groin vault and can be explained using the stiffened arch analogy. The forces from each individual hypar are carried down to the groins, which then take the weight of the structure as if they were axially loaded three-hinged arches, owning to the shell thinness at the crown of the structure behaving like a hinge (Billington, 1982). For a more detailed description of this behavior, the previous chapter can be consulted.

One example of a circular array of hypar shells is Candela's La Jacaranda Nightclub, a structure made of three half-saddles projected onto triangles that contains free edges left open to the air (Diaz, 2010). The roof is structurally interesting in that it has an odd number of shells,

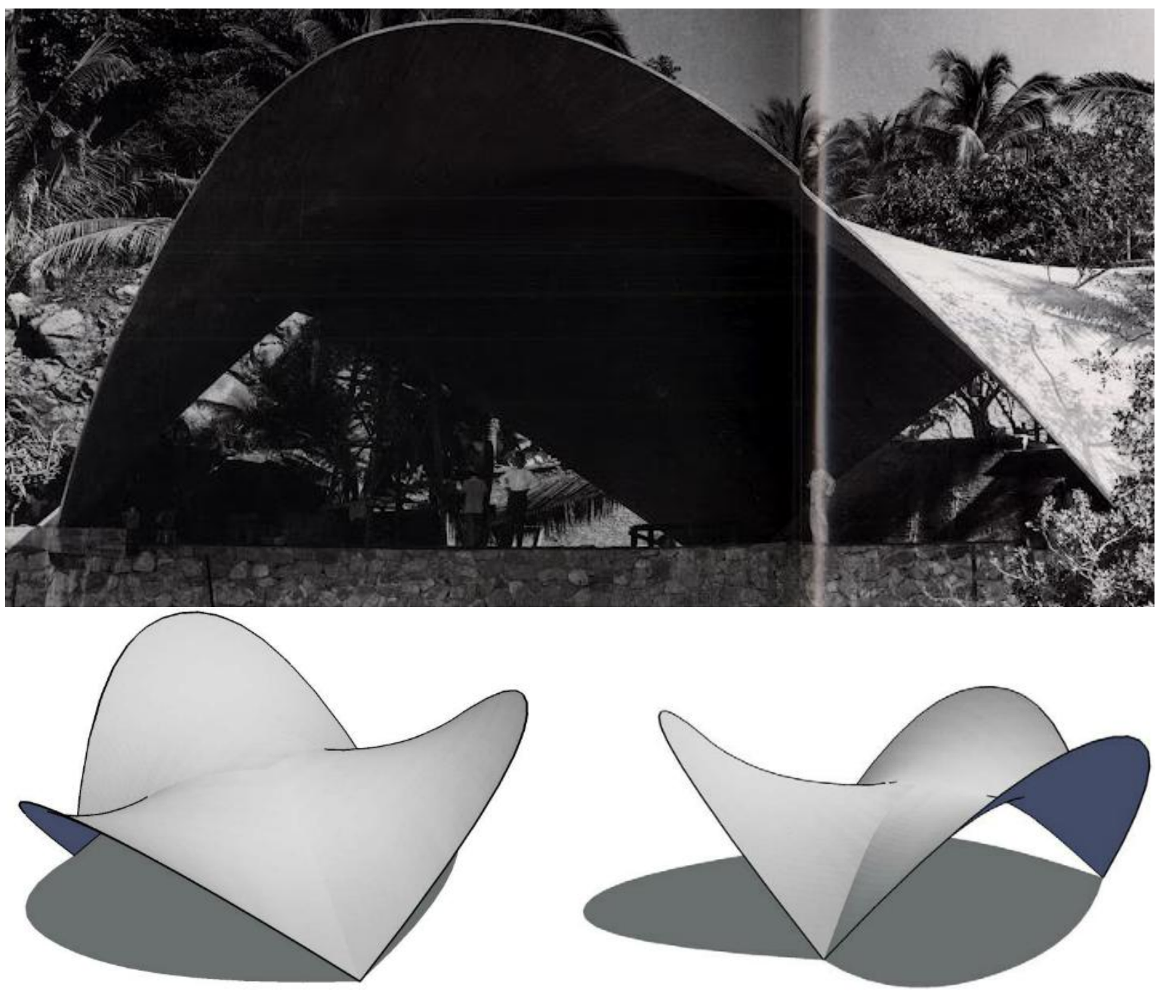


Figure 4.10: Images of Candela's La Jacaranda Nightclub, showing the irregular appearance of the shell despite being a polar array of hypars. Images courtesy of Julio Diaz.

preventing the groins from forming continuous arches. Despite this curious difference, finite element models have shown forces to flow axially through the groins just the same, causing the greatest stresses in the lower groins in a manner similar to even-number configurations (Diaz, 2010). The asymmetry of the tri-shell shape does produce a disorienting visual effect however, since it is difficult to completely conceptualize the radial nature of the structure from a singular vantage point (See Figure 4.10). From the perspective of the photo, the shell appears to the untrained or passing eye as an irregular geometry, which in this case works to produce a positive architectural effect. La Jacaranda was built in the gardens surrounding the Presidential Hotel in Acapulco, serving as a cabaret on the water for hotel guests (Diaz, 2010). Due to its configuration of only three shells, the roof is able to have a large opening directly facing the sea, while the other openings slink into the background greenery of the garden. A view from the hotel roof makes the strange geometry look like a tortoise thrown up on the shore by an angry tide, an architectural expression of its beachfront context (Diaz, 2010). Functionally, the shell's design fits its surroundings perfectly, creating a breezy, open atmosphere for dancing, dining, and a full bar (Diaz, 2010). Unfortunately, the shell was knocked down after only two decades of use, but during its lifetime it was an innovative and successful manifestation of a group of intersecting hypars arrayed around a central point.

The most famous circular groin array of hypars is Los Manantiales Restaurant at Xochimilco, also designed by Candela. The structure is made of four full intersecting saddle hypars, leaving eight soaring edges radiating out in thin concrete from the restaurant's center (Garlock *et. al.,* 2008). The restaurant's shape, reminiscent of a flower, fits perfectly with the surrounding floating gardens, although this is purely coincidental because the structure was originally planned for another site (Garlock *et. al.,* 2008). The elegant free edges of the shell represent an advanced structural design by Candela, who worked hard to ensure that his later structures could visually express their true thinness. On the interior, although the structure contains glass curtain walls, the thin vaults and large openings allow for substantial natural light and views of the surrounding gardens, making for a pleasant dining experience. In terms of combining thinness, structural efficiency, economy, and aesthetic sensitivity, Los Manantiales Restaurant has few real rivals (Garlock *et. al.*, 2008). Along with the Jacaranda Nightclub, the restaurant illustrates how a circular array of hypars can be structurally and architecturally successful for programs that require large vaulted spaces with openings in each direction.

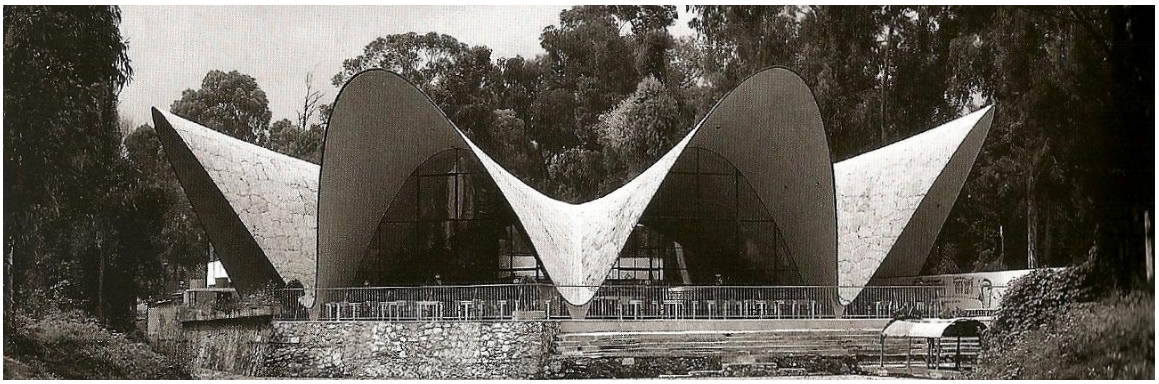


Figure 4.11: Los Manantiales Restaurant by Candela. Photo courtesy of the blog *Contento y Concepto*.

The next two categories of hypar configurations are also closely related, just as the circular array represents a special case of the groin vault family. Each of the next two categories is made of hypar umbrellas, themselves the product of many individual tympans. However, this thesis chooses to make a distinction between umbrellas that are free-standing and umbrellas that are continuously connected. The former category is useful for open air structures that require protection from rain and sun, but not much else. The latter category of continuously connected umbrellas amounts to a more efficient form of slab and column construction, since fields of hypars tessellated over a plane can be made to cover a closed warehouse or subway station. The structural behavior of the two categories is also different, since a freestanding hypar umbrella is a pure cantilever, but adjacent connected umbrellas can brace each other against deflections. Unfortunately, many existing structures blend the distinction between these two categories: for example, the Maimi Marine Stadium is an entirely open-air structure made of asymmetrical umbrellas connected along only one axis, whereas Félix Candela's Rio Warehouse uses this same principle but tessellates groups of umbrellas in both dimensions, leaving a mostly closed warehouse with only gaps in the walls and between tilted umbrellas for windows. Despite this ambiguity, it is important to make a distinction between mostly freestanding and large fields of connected umbrellas, because in the majority of cases the architectural uses for each category are entirely different.

Candela built numerous freestanding umbrella structures all over Mexico, which helped support the designer financially and allowed him the freedom to experiment with more daring designs (Garlock *et. al.,* 2008). A freestanding umbrella is a cheap, structurally efficient roof for many shading applications (See Figure 4.12). As opposed to straight slabs, the curvature of the

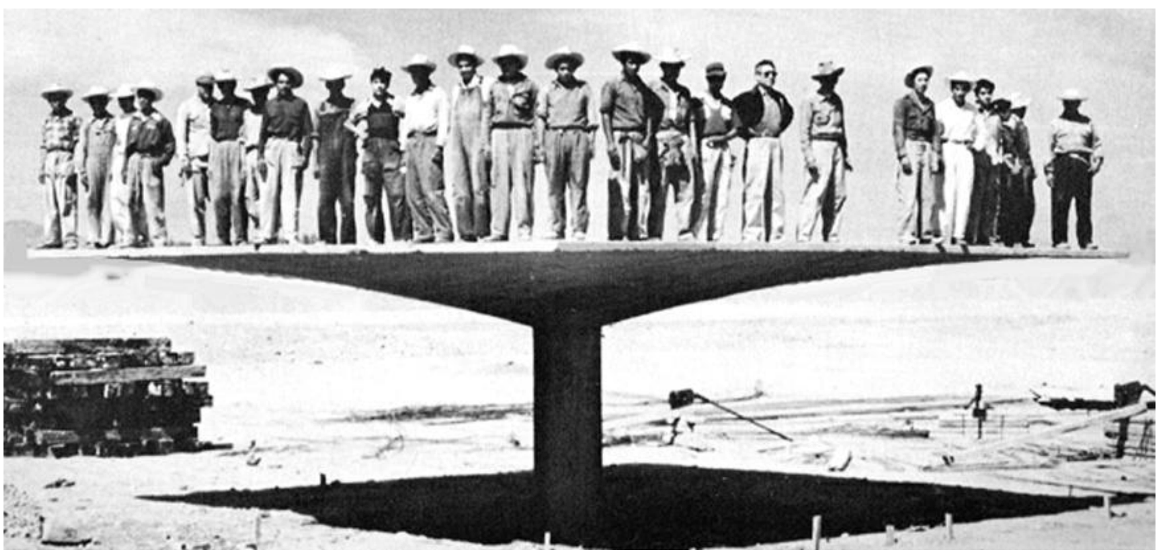


Figure 4.12: An unidentified example of a Candela Umbrella. Photo courtesy of *Subtilitas.*

umbrella's tympans make the forces flow to the edges and folds, minimizing stress throughout the shell and allowing for umbrellas to be much thinner. The curvature of the roof creates a natural bowl which provides a complication in the case of rain, but many of Candela's umbrellas solved this by having a drain pipe installed in the column (Garlock *et. al.,* 2008). Unlike vaulted hypar structures, umbrellas are not useful for applications that require unobstructed interior space, since an umbrella's central column forces designers to plan the floorspace around this obstacle. In the example of the Miami Marine Stadium, the placement of the columns in the stands of the stadium led to a considerable disagreement between the architect and engineer, as is explained later in Chapter 5. However, despite the impediment of the column, a free-standing umbrella could be effectively applied to a variety of small architectural demands: the roof of a stadium grandstand, a bus-stop, carport, shed, or building entrance, to name a few.

When umbrellas are connected and either tessellated in two different directions or closed off by other building elements, the architectural uses and structural behavior of the form can be different. The resulting field of umbrellas produces a continuous roof supported by columns, common to almost all buildings made from traditional concrete construction (See Figure 4.13). The advantage of having hypar umbrellas is the more efficient transfer of loads down to the

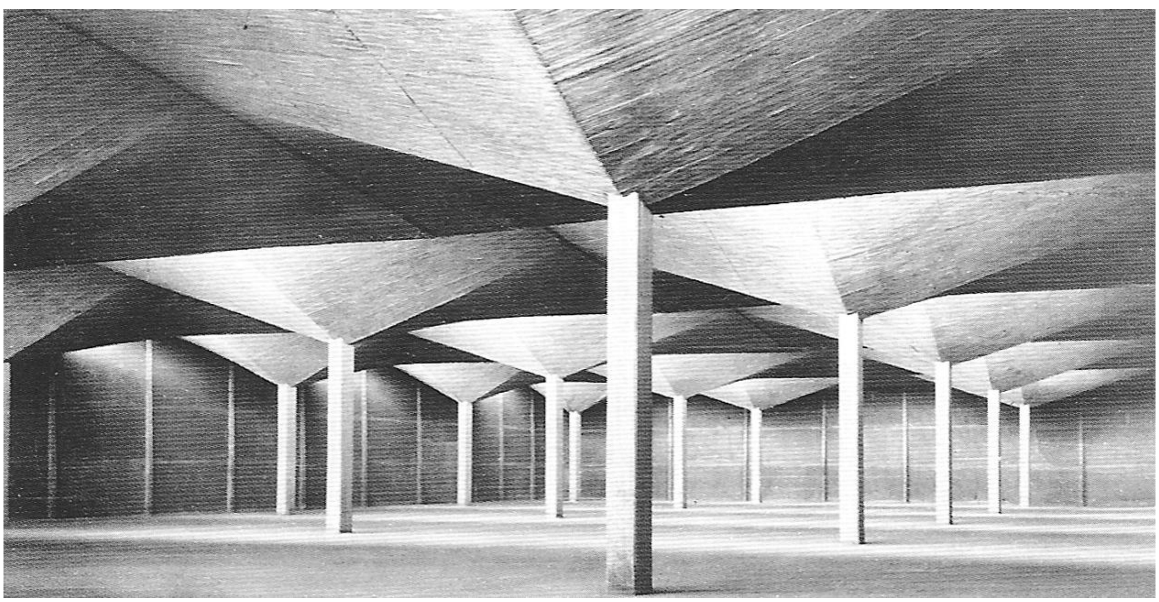


Figure 4.13: A field of continuous umbrellas. Although the geometry appears to be made of lines instead of curves, the effect of a field of hypar umbrellas is still conveyed. Photo courtesy of Diego Terna.

column, which takes place in the curved folds of a hypar. For various reasons, continuous umbrellas can also be skewed or tilted in a certain direction, leaving gaps in the roof for light and air. If this design alteration to the standard umbrella is made, the umbrellas behave structurally like freestanding cantilevers along one axis, but the connection between adjacent shells in the other axis helps stabilize each shell (Adriaenssens *et. al.*, 2012). Despite these gaps, the architectural uses of a continuously supported roof stay the same—the roof just happens to be made of strips. Félix Candela designed numerous examples of tilted umbrellas, arranged in strips to cover the space of a market or factory. The Coyoacán Market, the Anahuac Market, and the High Life Textile Factory are all good examples of this configuration (See Figure 4.14). Umbrellas are one of the most extensively used hypar forms by Candela, because they are an easy to build and architecturally versatile hypar configuration. The umbrella has been called his "bread and butter", because it is a standard form that can be replicated easily with reusable formwork, a property that makes the configuration economic as well (Garlock *et. al.* 2008).

Figure 4.14: Rows of titled umbrellas in the Anahuac Market. Photo courtesy of Jorge Ayala.

Another main configuration of the hypar form is the gabled roof, which is made of four tympans connected by their in-plane edges rather than their slanting ones. Gabled roofs are flatter in profile than a groin vault, and they do not create as much vaulted space. Nevertheless, since they closely resemble a normal gabled roof and can be made easily from straight line generators, designers have used them for a number of applications. In terms of behavior, gabled roofs show that for some configurations, membrane theory does not closely match reality in the analysis of hypars with straight edges (Billington, 1982). For this reason, it is important that structural designers do not rely only on the membrane theory without comparing it to analogies, testing physical models, or supplementing it by bending or more rigorous analysis. In a gabled roof, membrane theory predicts the interior flat ridge beams carry compressions that balance out in the middle, and the exterior slanting edges of the shell carry pure compression to the supports. However, more precise analysis shows that in actuality the flat ridge beams do little but add dead weight to the structure, introducing substantial bending and vertical deflections into the shell (Billington, 1982). A more realistic conception of the gabled roof treats it as two flat intersecting diagonal arches, an analogy which adequately takes into account the interior deflections. The interior deflections cause relaxation of the tension thrust, and large compression stresses shoot straight down the shell diagonals into the supports (Billington, 1982). Due to this phenomenon, the actual compressive stresses in a shell are double what are predicted by membrane theory (Billington, 1982).

The complicated behavior of hypar shell gabled roofs has lead to innovative, sometimes piecewise structural solutions. A good example of a gable roof is Anton Tedesko's Court House Square extension roof in Denver, built in 1958 (See Figure 2.3 on p. 19) (Garlock *et. al.*, 2008). In this structure, it is noteworthy that the overall shape was designed by famous architect I.M. Pei and dictated to Tedesko, forcing the design process to include compromises between architect and structural engineer. In his initial sketches, Pei called for an entirely smooth, ribless shell. From experience, Tedesko knew that the flat portions in the top of the shell would buckle, and thus he added two strongly reinforced slab bands running along the roof's flat central ridges (Garlock *et. al.*, 2008). With the addition of slab bands and Tedesko's careful management of creep, the roof had no major structural problems (Garlock *et. al.*, 2008). The example of the Denver Court

House square roof shows that although the hypar gabled roof is not among the most intuitive or structurally efficient configurations of the hypar form, it can be made to work. For a successful hypar gabled roof, the designer must understand its behavior as diagonal compression arches, employ a sufficient curvature at midspan, and stiffen the flat central part and supports with light ribs that guard against buckling, creep, and displacements (Billington, 1982).

The double cantilever, which behaves similarly to a cantilevered umbrella, makes up the next distant configuration for the hypar form. A double cantilever is a saddle projected onto a rhombus, leaving four straight edges, two soaring points, and strips of increasingly smaller arches running from middle supports out to these points. The double cantilever is often used to form a single, curving roof of a building. As such, its openings are usually covered with walls, glass curtain walls, or even load-bearing columns when live loads require large, heavy stiffening beams on the edges (Billington, 1982). Each of these features can obscure the structural performance of the form, since a building can look like a curved roof sitting on walls rather than a cantilever held up by central supports. However, since all forces flow back to the two main supports, a double cantilever roof leaves the architectural options of having covered, open areas for a porch or entrance, or interior space enclosed by a curtain wall (Billington, 1982). This versatility within the context of traditional architectural forms allowed the double cantilever to branch into building uses not touched by other hypar shells. For example, many single family houses have used the

Figure 4.15: Double Cantilever hypar roofs on residential and commercial buildings. Photo courtesy of *Flickr Commons*.

double cantilever configuration (although not always in thin shell concrete), as have hotel lobbies, offices, and other relatively small buildings (See Figure 4.15). Slightly larger buildings, such as the Predeal Railway Station in Romania, have used the form as well.

When employed in a thin shell, the double cantilever exhibits an upside-down version of the structural behavior of a hypar umbrella. The double cantilever's analogous structural form would be a balanced, two-sided cantilever, in which the transverse crown carries longitudinal tension and the straight edges carry axial compression to the supports, so that the total moment of the cantilever is resisted (Billington, 1982). A number of structural complications occur from this form, the first of which is the need to prevent local bending failure in the shell near the tip. In this region, the length of the moment arm between the longitudinal crown and the edge beams is decreased, and the designer needs to be careful to make sure that the shell is able to adequately resist bending and compensate for the smaller moment arm. Good examples of successfully employed double cantilever hypar forms in the United States still exist as part of the entrance for the Sacramento Zoo (*Dwell*). This set of hypars gives a direct comparison between open air double cantilevers and ones that have been filled in to create a traditional building (See Figure 4.16). However, each structure retains soaring, pointed edges, while architectonically communicating the function of the massive supports by covering them with masonry.

Figure 4.16: Double Cantilever hypar roofs on the entrance to the Sacramento Zoo. Photo courtesy of *Dwell*.

The second to last configuration of hypar shells, the tower, is a special case because it is used predominantly for the walls of cooling towers rather than the roof of a building. Instead of the hypar being placed horizontally as in most configurations, it is instead placed vertically and revolved around an axis. In other words, it is as if a vertical cylinder was twisted at the top and bottom and is now thinner in the middle than at the ends, although hypar towers are generally made wider at the base than at the top (see Figure 4.17). Since these forms are used to hold liquid and are subjected to entirely different loading than shell roofs, they behave as though they are a cone retaining wall (Billington, 1982). Membrane theory approximations for this type of hypar shell are complex, because the tower has a variable thickness and is reasonably subjected to gravity loads, internal pressure, wind loads, and sometimes seismic loads (Billington, 1982). Despite complications associated with calculating loads in the tower, towers were one of the first manifestations of the hypar form to be built, appearing by 1930 in Liverpool (*Popular Mechanics*, 1930). The shape of the shell allows for enormous towers to be built for the purpose of cooling

Figure 4.17: Hypar cooling towers at a nuclear power plant. Photo courtesy of *Electric Tree House*.

water in power plants, and these early hypar towers were much larger than rectangular or even cylindrical towers could be made at the time. Thus, the double curvature of the hypar shell gives it additional strength, and in an aesthetic sense, a curved tower can form a striking profile when viewed from the ground standing alone or as part of a large installation, such as in a nuclear power plant.

Of course, there are a number of innovative structures that employ the hypar form in such a way that it does not fit neatly into one of these categories. The last named category, called the hybrid, is meant to capture these structures, would could have any number of hypar forms. Usually, they involve distinct elements of more than one spatial category. One of the most advanced examples of a hybrid hypar structure is Félix Candela's Church of Our Lady of the Miraculous Medal, built in Mexico City in 1955 (See Figure 4.18) (Garlock *et. al.,* 2008). According to Candela, he formed the church out of asymmetrical umbrellas, tilting one edge to the ground and placing the other edge adjacent to another mirrored umbrella. Next, the lower side was conceptually 'folded up', leaving an open archway for a window on the side of the

Figure 4.18: Milagrosa from above, showing the bays of tilted, warped, hypar umbrellas. Photo courtesy of Yadariz Ramos.

church (Garlock et. al. 2008). Multiple bays of these umbrella pairs are arranged in ascension towards the main apse, which is formed by a much larger hypar with no middle support. For good measure, Candela added a side roof with eight smaller hypars, as well as a lazy-S folded plate roof over an adjacent chapel (Garlock *et. al.*, 2008). The geometric complexity of the form is stunning, and leads to both visual beauty and difficulty in analysis. Although the original shape may be a cantilever umbrella, its form is so warped into a mixture of arches and gables that it does not act according to one simple analogy. There are a number of other hypar structures built to this level of geometric complexity, but it makes little sense to try and fit them into a single category. However, as brilliant examples of hypar shell designs, they must not be forgotten.

The nine different configurations show that the hypar is a highly versatile form structurally, spatially, and architecturally. Given its propensity for structural efficiency, ease of construction, resource economy, and architectural success, the hypar form is worth exploring in the contemporary fields of engineering and architecture. Even though the hypar has historically been used in so many ways that the form may seem exhausted, Candela and other designers have shown its spatial possibilities are virtually limitless. Diversity within categories, as well as a variety of hybridized shapes, should encourage future designers to come up with configurations that have not been done before yet are an elegant solution to a particular set of design constraints. History can also serve the field of design in a more direct way than proving versatility, however. The configurations explained in this chapter provide a basis for understanding the past successes of hypar shells, which can in turn be used as a reference point in the development of new structures. A classification system such as this one is important because it organizes the form based on spatial configurations designed in response to a particular set of design constraints. Classification allows a structural designer or architect to efficiently and systematically gain knowledge of what has worked before when he or she faces similar constraints.

4.3. Design Trends

Throughout the study of hypar shell examples, a number of design trends have been found to cross categorical boundaries. These trends are important in terms of their own effects and how they relate to other trends. A visual mapping of trends is also illustrated in Figure 4.1, which uses colors to highlight relationships.

The first of these trends is inversion, which will be considered the conceptual shift from defining a surface by its vertexes to defining it by its points of interruption. The initial step in hypar inversion is the cutting straight lines into tympans. Multiple hypar pieces are then combined at these lines, so that an inverted structure has definitive creases rather than continuous curves. The creases can be understated in umbrellas with low slopes, or they can be folded to the extreme, as in La Candelaria Subway Station (See Figure Figure 4.19) (Garlock *et. al.*, 2008).

Figure 4.19: The folded typmans of Candela's subway station. Photo courtesy of *Wikipedia Commons.*

In inverted hypars, the forces tend to flow through these creases, and they become the portion of the shell that dictates the overall shape. In mathematical terms, it is analogous to going from a parabola, which is defined by the curve itself, to a hyperbola, which is defined by its asymptotes. An inverted hypar is usually referred to as a hypar umbrella, and these umbrellas can have a number of uses. The hypar umbrella can be employed in buildings that have a program with a traditional, rectilinear footprint, such as a warehouse or a subway station, because they closely resemble normal slab and column construction. In virtually all cases, the umbrellas must sit on top of columns, and their straight edges can be combined to fill out a rectangular space. In this configuration, they provide the same type of column-supported, slab-covered space, but have a different aesthetic effect and are more efficient with material than traditional concrete structures (Garlock *et. al.,* 2008). Inversion is also useful for applications that require one or more free sides with a defined edge. Since inverted hypars generally take the form of cantilevered tympans branching from a central column, it can easily end in a straight edge, which defines the extent of a building's footprint. At this point, the structure can be left to the open air (common in stadiums or Candela's structures in warmer climates) or walled off with a glass curtain or traditional wall.

Another major design trend is the use of tessellation, which is especially useful in hypar shells because it allows for the reuse of formwork and can lead to a drastic reduction in costs. It has been estimated that formwork can contribute between one third and two thirds of the total cost for a reinforced concrete structure (McCormac *et. al.,* 2009), making the reuse of formwork an attractive technique for builders. In a tessellation, one simple form is used over and over again to generate the final shape of a structure. Tessellations can be either Cartesian or polar, meaning that the tessellated elements can be translated and placed side to side, or then they can be rotated and placed in a circle. It is also possible to tessellate elements vertically by placing them on top of one another, but this is not seen in many hypar examples because a curved umbrella or vault does not make as good of a floor as it does a roof. In some forms of tessellation, all of the elements work together and are required for the structure to function. An example of this is the Los Manantiales Restaurant, which would not work if one of the saddles were removed. On the other spectrum is the Bacardi Rum Factory, which tessellates self-sufficient structures. If one of these elements is removed, the factory would simply be smaller. In either case, tessellation has shown to be an effective technique for reducing material costs in both large and small structures, and it would be compatible with certain advances in shell technology such as prefabricated or modular formwork.

The next design trend is vaulted space, characterized by buildings that require large spans or high-volume interiors. Most of the major examples have either inversion, which lends itself to columns, or vaulted space, which is a series of extruded arches. Often employed in a saddle or groin vault shape, a hypar revolved halfway around a horizontal axis can create a system of light, thin arches which behave simply and create clear, unobstructed interiors. Common characteristics of vaulted space include smooth walls that blend into the roof, high ceilings, the transmission of the acoustical affect of specularity (Moussavi, 2009), and the efficient diffusion of light. This third property is especially useful in terms of sustainability, since building footprints allow for ample daylighting and there are no sharp corners in vaulted space to block light. Lighting fixtures installed in the concrete roof can be used to supplement the natural light entering a vault through its open end. Ultimately, a decrease in the overall lighting power density caused by the shape of the building can decrease energy costs (Zigo, 2011). As shown by this chapter’s various hypar examples, the advantages of the vaulted interior space formed by horizontally placed hypar shells have been used for a variety of different architectural programs, such as chapels, restaurants, factories, and nightclubs.

A number of hypar shell examples also contain glass curtain walls, which are used as a non-structural barrier between interior and exterior space. In terms of form, curtain walls are often employed in structures with free edges to normalize the footprint of the building into straight lines, such as in the Manantiales Restaurant, or to delineate where the roof ends, as shown by numerous double cantilever examples. Glass can also be used to connect tessellated shells to form a continuous surface, which Candela did both in his Bacardi Rum Factory and St. Vincent de Paul Chapel (See Figure 4.20). The curtain wall trend is useful for any climates or programs in which the occupants require a type of environmental conditioning. Although

Figure 4.20: An early photograph of Candelas San Vicente de Paul Chapel. The structure now has stained glass in between the hypars and glass curtain walls below them. Photo courtesy of Octavio Domosti.

neither glass nor thin shells are good insulators (unless insulation is built into the shell), having a sealed off environment is still preferable in some cases to an open air structure. A glass curtain wall is the best way to do this and not lose the natural lighting let in through the large openings of many shell forms.

Another trend seen throughout the study of existing hypar shells is the use of asymmetry, which has been applied in different ways to generate a variety of effects. In the saddle configuration, since the curves defining the shape are parabolas and get steeper away from the vertex, a degree of asymmetry can create a soaring peak. This peak will have a different scale and slope than the rest of the shape, but still follow normal structural behavior. A soaring peak has traditionally contributed to the aesthetic success of religious buildings, such as the Chapels Lomas and St. Vincent de Paul. As noted in *Félix Candela: Engineer, Builder, and Structural Artist*, the Chapel Lomas "supports itself and has the dramatic curvature, striking size, and indication of thinness that represent the frontiers of the form" (Garlock *et. al.* 2008, p. 128).

Thus, asymmetrical thin shell structures accomplish in relative scale the dramatic height desired in traditional cathedrals without the massive amount of material required by designs of years past.

Asymmetry can also be used in umbrella tessellations to produce innovative effects involving visual form, lighting, and structural performance. The Miami Marine Stadium is one example of where this occurs, since the front side of the umbrella is much deeper than the back side, and it also comes to a flat edge rather than a peak. This asymmetry, coupled with supports in the middle and back of the stadium, creates a large cantilever which covers spectators in the seats below while offering an unobstructed view of the action in front of them. Candela also uses asymmetrical umbrellas to let natural light into a number of his structures. In both the Rio Warehouse and the Coyoacan Market, rows of tilted umbrellas are built right up against each other so that the gap between the lower edge of one set of umbrellas and the higher edge of the next set forms a vertical opening for the placement of windows (Garlock *et. al.,* 2008). These examples of asymmetry show that its uses are diverse and its effects are wide-ranging.

Although it is mostly a trend driven by aesthetics and a desire by designers to have clear structural expression, free edges also qualify as a design trend. Free edges contain no stiffening brace or arch along the outsides of the structure and allow for the true thinness of the shell to be visually expressed. This trend is closely linked to two other trends mentioned in this section: glass curtain walls and irregular footprints. In a saddle form free edges create an elliptical edge, meaning that the roof must be left open, or the usable floorspace must be reduced by the presence of glass. Thus, there is a choice to be made by a designer of a free edge shell regarding the importance of having a sealed building versus an open air structure. A class curtain wall can waste a considerable amount of space still covered by the edge of a shell, as illustrated by Los Manantiales Restaurant. If a shell is in a warm climate, it may be more advantageous to leave the structure open to the air, such as in the Jacaranda Nightclub. A double-cantilever shape does not pose this problem of being forced into a choice between glass and an irregular footprint, since its free edges already form straight lines and curtain walls can simply follow a slightly offset outline of the roof. In a variety of configurations, free edges were one of the last frontiers occupied by shell designers, believed to be the truest form of architectonic communication in the field.

The last design trend mentioned in this classification system is the irregular footprint. This trend is not as much of an intentional design goal as it is a side effect of building curved forms with free edges, and as such it is dealt with in a variety of different ways. The solutions to irregular footprints have already been discussed in relation to other trends, and it would be redundant to give them their own discussion here.

Knowledge of design trends in hypar shells and their relationships with each other can inform the design process for a shell designer who is addressing programmatic or structural problems that are solved in a specific way. Each trend is responsible for a variety of different structural, visual, and in some cases even environmental effects. Many of these effects were not intrinsically known or desired by designers in the earlier stages of their careers, since they were observed after construction or developed through experience. The design trends here represent aspects of shells that were generally successful; otherwise, they would not be trends. For example, Candela one tried embedding glass bricks inside the roof of a tessellated umbrella structure in Mexico, but the direct sunlight entering through the roof made the interior insufferably hot, and this was not attempted again (Garlock *et. al.,* 2008). Thus, these successful design trends are the result of cumulative knowledge gained during long careers in structural engineering, and are certainly valuable in light of their past success.

4.4. Sustainability of the Hypar Form

A property of the hypar shell's form that is becoming increasing relevant in the current building context is its ability to limit the use of materials during construction. There has been a considerable amount of research done into the amount of energy used in the production of typical building materials, and these values can serve as a starting point for understanding why material reduction is important to the environment. In addition to emphasizing the need for efficiency, an estimate of a material's embodied energy can be used to directly quantify a structure's environmental impact. For the purposes of this thesis, proof that concrete can be used sustainably, as well as evidence of a measurable difference in material use between hypar forms and regular concrete forms, is crucial. Without either of these two elements, the argument for hypar concrete shells as viable and sustainable design solutions in a contemporary context falls apart. In other words, concrete must be a widely available, heavily used, and reasonably energy efficient building material for hypar shells to be relevant.

The process of producing concrete (and one of its main ingredients, cement) has become a focal point of energy research due to its growing popularity as a cheap and accessible material around the world. According to the World Business Council for Sustainable Development, concrete is the most widely used material on earth except for water, with nearly three tons used annually for each man, woman, and child on the planet (WBCSD, 2005). To meet this demand, the cement industry produces about 5% of the world's total man-made CO_2 emissions (Battelle, 2002). Although this production is distributed unequally in terms of volume and efficiency, it is possible to calculate the amount of emissions caused as a direct result of making a certain amount of concrete. This value can change drastically depending on the region; for example, since China produces roughly 37% of the world's cement and does so mostly using inefficient, decentralized, and outdated technology, the carbon emissions due to concrete production in that part of the world are higher than in North America or Europe (Battelle, 2002). Nevertheless, emissions per volume values form one basis for quantitatively calculating the reduced environmental impact due to the more efficient structural design of concrete hypar shells.

Before moving too far, it is necessary to identify whether or not concrete is an environmentally friendly building material in the first place. According to Struble & Godfrey (2004), an evaluation of a material's sustainability must consider the production of material, construction, life-cycle, and demolition of structures built using that material. Taking all of these into account, Struble & Godfrey compare a traditional reinforced concrete structural design to a steel design for the same purpose and loads to determine which has less environmental impact. Their study shows that in general, concrete requires much less energy and has a lower net environmental impact than steel for a similar structural application (Struble & Godfrey, 2004). This is true for a number of reasons, most notably because the production of concrete can use recycled materials or byproducts of other manufacturing processes as aggregate and admixtures, reducing its embodied energy (Struble & Godfrey, 2004). Concrete structures are also easily demolished, and many products associated with concrete construction can be recycled, such as the formwork or even the concrete itself. Thus, of the two main construction techniques used by structural engineers, reinforced concrete is the more environmentally friendly choice for many applications. Although other materials such as timber may be more sustainable or advantageous for specific areas, concrete is a cheap, accessible, and has a reasonably low environmental impact for use in structural applications around the world.

A good estimate for the amount of CO_2 released into the atmosphere per cubic yard of concrete produced in the United States or other Western Hemisphere country is 400 pounds (Nisbet *et. al.*, 2003). This value is roughly equivalent to the amount of carbon released by consuming 16 gallons of gasoline in a car, or using a desktop computer or microwave for a year (Conniff, 2005). Given these estimates, coupled with the fact that concrete is often the best building material available, there is a clear environmental incentive to produce efficient structural designs that use less concrete to serve the same purpose. In order to evaluate the degree to which higher efficiency thin shells can reduce the environmental impact of a structure, the estimate of 400 pounds per cubic yard of concrete is applied to models of existing Candela thin shells. The emissions estimate is then applied to models designed with typical concrete construction techniques but similar spans and floor areas to these existing models. A detailed explanation of this exercise follows in the next few sections.

4.4.1.Model Methodology

Using dimensioned drawings available in published works about Félix Candela (Garlock *et. al.,* 2008) as well as the archives of Princeton's Civil and Environmental Engineering department, six separate models of different hypar shell configurations were modeled using the design software Rhinoceros© (Rhino) (see

Figure 4.21). For comparison purposes, three models of similar dimensions to these shells were also created using preliminary sitecast concrete size guidelines for the following types of construction: two-way waffle slab, two-way flat plate, and one-way solid slab (Allen *et. al.*, 2012). For each of these models, Rhino is able to calculate the floor area of the building, the surface area of the concrete structure, and the volume of any additional concrete components. From these values and the reported thicknesses of the concrete shells, the amount of concrete used in the construction of each structure was manually calculated and then converted to pounds of CO_2 emissions.

This thesis proposes the following metric for analyzing the environmental impact of thin shells:

$$Structural\ Efficiency\ Impact\ Ratio = \frac{Amount\ of\ Carbon\ Emissions\ Due\ to\ Material\ Production}{Area\ of\ Usable\ Floor\ Space}$$

Although this ratio involves many simplifications, it provides a good standard for comparing traditional concrete construction with thin shells across many different shapes and building sizes. There is considerable precedence for the practice of using material used per coverage area as a measure of structural efficiency (Dischinger, 1928), but this formula seeks to take the ratio one step further, converting the amount of material to a measurement for environmental impact. Essentially, the formula is a quantitative measure of the negative environmental impact due to material production incurred per footprint size of the structure, and it seeks to use the measure of floor space as an equalizer between large and small structures. Structures with smaller Structural Efficiency Impact (SEI) ratios may vary in size, but they span their useful areas more efficiently than structures with higher SEI ratios. Since SEI does not directly cover differences in span lengths, and roof thinness, these aspects of the models will be discussed separately.

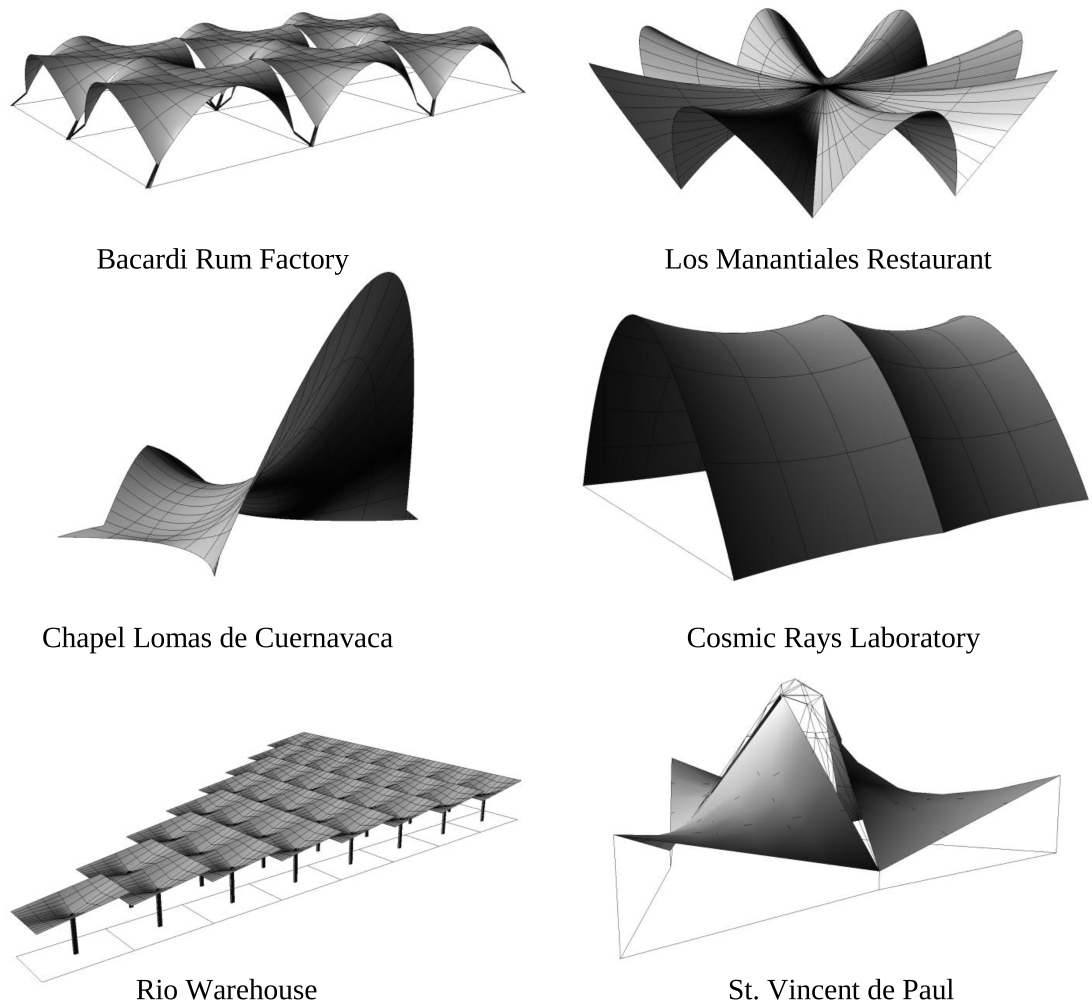


Figure 4.21: Renderings of the six rhino models generated for material calculations.

There are a number of limitations to this method, which result from the limited scope of the study. The environmental impact assessment does not take into account other materials used in the construction of these shells, such as glass or masonry. It also does not address the impact of the formwork used. In the Candela examples, the formwork was made of wood (Garlock *et. al.,* 2008), although metal or other types of formwork can also be used in thin shell construction. Sometimes this formwork is recycled, but other times it is placed in a landfill, leading to additional waste (Struble & Godfrey, 2004). However, this thesis assumes that the difference in formwork's contribution to environmental impact will be negligible compared to the huge difference in amount of concrete, considering both concrete building techniques require formwork. The amount of material used in the foundations of the structures was also ignored, but this is not assumed to change substantially for the different types of construction.

4.4.2. Sustainability Evaluation Results

The calculated SEI ratio for each of the studied models is presented in Table 4.1 and Figure 4.22, while the raw calculations can be found in Appendix A. This simple comparison clearly shows that each hypar shell performs substantially better than the three more traditional modes of concrete construction. On average, the equivalent traditional slab and column structures use more than 4.5 times as much concrete as the Candela shells in this study. Amongst the shells themselves, there is a slight variance, but since there is a small sample size these can be explained individually. For example, the shell with the largest ratio, the Cosmic Rays Laboratory, was one of Félix Candela's earlier structures, and he did not yet have the experience to be an aggressive designer. As such, he included three 31-cm thick stiffening arches in the design to support the shell, and these arches contain a considerable amount of concrete (Garlock *et. al.*, 2008). On the other end of the spectrum, the shells with the smallest SEI ratios, the Rio Warehouse and St. Vincent de Paul Chapel, also have special characteristics that contribute to their efficiency. The Rio warehouse is the only structure made entirely of umbrellas, while also being open to the air on its outside envelope. These two factors make it distinct from the vaulted shells that bring the edge of the shell the whole way to the ground, creating a higher ratio. In the case of St. Vincent de Paul, the design uses mullions and glass to connect three separate concrete roofs. If this entire glass section was instead made of concrete, the ratio would surely be higher, but as is, the chapel uses the least amount of concrete of the structures studied. Nevertheless, despite these differences and peculiarities, the models in this section represent a diversity of examples across both the categories of hypars and the category of traditional slabs. Across the board, hypar shells are shown to use concrete much more efficiently.

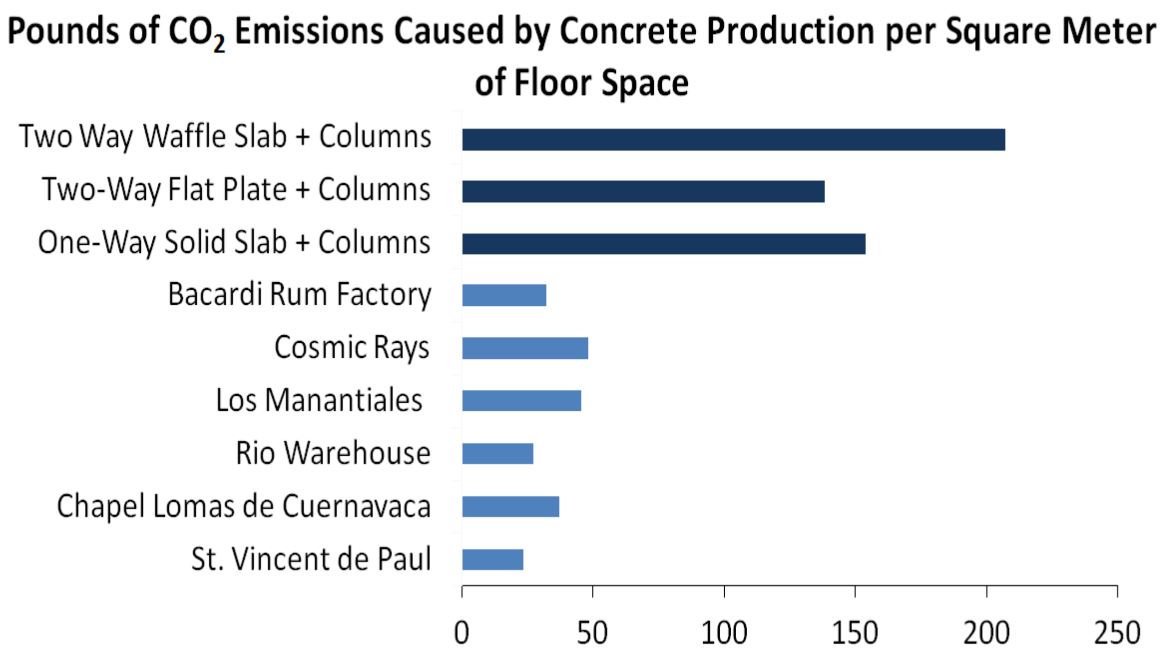


Figure 4.22: Results of the Structural Efficiency Impact Ratio calculations.

Table 4.1: Structural Efficiency Impact Ratio and Span Lengths for Models

Model Name	**Largest Span**	**Smallest Span**	**SIR**
	(m)	(m)	(lbs. CO_2 / m^2)
St. Vincent de Paul	29.0	18.1	23.62
Chapel Lomas	31.0	18.0	37.05
Rio Warehouse	14.6	9.6	27.42
Los Manantiales	42.4	5.8	45.33
Cosmic Rays	12.0	10.3	48.16
Bacardi Rum Factory	31.0	26.0	32.18
One-way Slab	6.1	6.1	154.07
Two-way Flat Plate	11.0	7.62	137.48
Two-way Waffle Slab	16.8	17.8	206.45

Although results of other building characteristics are less conclusive than the amount of material reduction measurements, the shell models also illustrate other advantages over traditional concrete construction. The most obvious advantage is span length, since hypar shells can be shaped to efficiently span much greater areas than rectilinear slab and column constructions. The increases in vertical and horizontal free space afforded by hypar shells allows for versatility, enhancing the usability of the structure. Figure 4.23 compares values for the largest and smallest spans measured in the various models. For the three traditional means of construction, these spans represent the maximum spans recommended as 'typical' according to preliminary concrete sizing guidelines (Allen *et. al*., 2012). In this study, a simple one-way solid slab has the shortest maximum span, followed by the two-way flat plate. Although the waffle slab allows for much higher maximum spans, five of the six hypar models easily surpass it. The larger spans are made possible both through the curvature of shell shapes as well as their lightness, and can be useful in many programmatic applications that require large, covered areas free of columns or other obstructions.

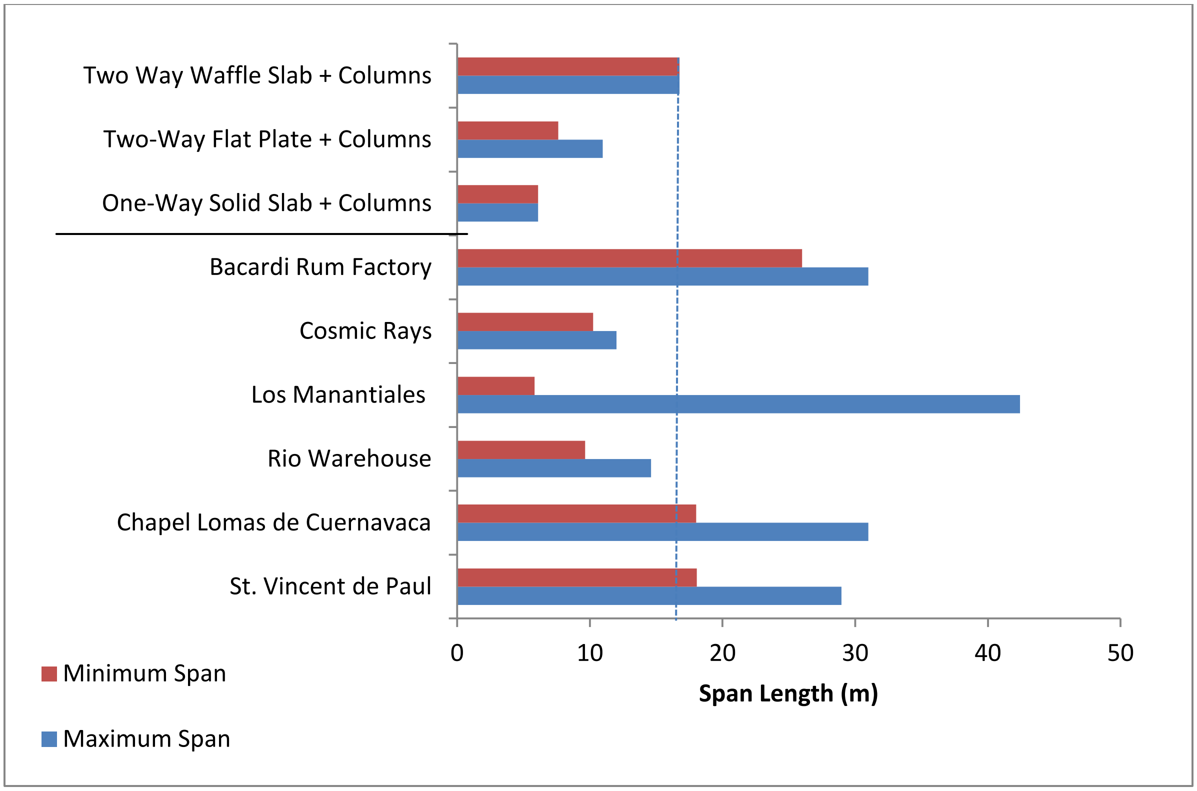


Figure 4.23: Roof span lengths for different models. In general, hypar shells have longer spans; the measurement for Los Manantiales is taken across a diagonal groin which may be misleading, but its circular array nevertheless leads to a large amount of free interior space.

As implied by their name, thin shells typically have much thinner roofs than normal concrete construction techniques, which can increase ceiling heights and add volume to a building. Of the models studied, the average shell thickness was 4 cm, while the average slab/plate thickness was over 25 cm. A simple comparison ignores the fact that since slab and column concrete structures are made of flat planes, they are often stacked, with one slab ceiling acting as the next level's floor. Stacking is not usually seen in thin shells, and each of the shell models studied here represents only one-level roof structure. However, there are plenty of applications in which continuous and cavernous interior volume is desired rather than stacks of floor space. For these structures, shells prove to be the more efficient option.

After modeling and a comparison of the generated data, it is clear than thin shells are able to use less material than traditional concrete construction methods in many applications. Due to the amount of embodied energy in concrete and the rate at which it is used as a building material throughout the world, the use of thin shells as an alternative to traditional concrete construction could have substantial environmental implications. When less concrete is used, less carbon is emitted into the atmosphere, leading to a smaller environmental impact. The scope of this study was restricted to only nine examples, which were chosen to be representative but are certainly not exhaustive. As such, further analysis would have to be completed to judge the larger scale effects of an extensive shell building movement. However, in terms of single structures, the emissions savings are significant in a direct comparison between shells and traditional slabs. In addition being environmentally friendly, hypar shell designs have longer spans and thinner roofs than more traditional alternatives. Although the curvature, lack of inherent insulating properties, and other unique characteristics of hypar shells may make them unusable for certain locations or purposes, shells are a quantitatively efficient and sustainable design option for architects and engineers.

5. Design and Construction of the Miami Marine Stadium

5.1. Introduction

When they were constructed in 1963, the eight curved hypar shells of the Commodore Ralph Middleton Munroe Miami Marine Stadium formed one of the longest cantilevers in thin shell concrete that had ever existed (Adriaenssens *et. al.*, 2012). The realized shell roof was a daring, progressive design, the product of considerable efforts by a visionary architect in Hilario Candela, an innovative engineer in Jack Meyer, and many skilled builders. Without the aid of complex mathematical analysis tools and restricted by a modest cost ceiling, the designers successfully created a sustainable reinforced concrete hypar shell structure accepted as an exceptional feat of both architecture and engineering. Today, after falling into disuse and having been neglected for nearly two decades, the stadium is recognized as one of America's Most

Figure 5.1: The Miami Marine Stadium in its current state. The stadium is for the most part structurally sound, but it is deteriorating and covered by debris and graffiti due to years of neglect.

Endangered Historic Places by the USA National Trust for Historic Preservation (Adriaenssens *et. al.*, 2012). Due to its architectural beauty, longstanding structural performance, and situation as a public building in an urban context, the Miami Marine Stadium has been selected in this thesis as a case study for the implementation of thin shell hypar structures. To gain a more thorough understanding of how the stadium came to be, it is necessary to explore the various constraints present on the project as well as detail how the designers of the stadium responded to these constraints with the resources available to them at the time. Historical knowledge of these constraints and responses are useful to the modern designer of shells in both an academic and practical sense, in that what has been done before can be repeated, improved upon, or simply admired as an innovation that is no longer necessary or possible.

This chapter supplements the finite element analysis made in the next chapter by giving context to the purely structural aspects of the stadium. First, a description of how the design was formulated through the back and forth efforts of the architect and engineer is given. In the words of Jack Meyer: "I made the stadium strong; Hilario made it beautiful" (Meyer, 2012). However, the actual tension between the demands of an architectural sketch and the realities of calculations was much more complex, and led to a series of compromises that retained the main goals (beauty and strength) of each respective discipline. Second, the construction methods is discussed, since the way in which a shell is built can substantially affect the cost and sustainability of a structure. Although much of the erection procedure was designed by the subcontractors responsible for building the stadium, Meyer was nevertheless intimately involved with its construction and can provide insight into the efforts necessary to efficiently build a thin shell concrete structure. Moving forward into the present, a brief review of the current condition and performance of the stadium is given next. By a pure definition, a sustainable structure must have a long life and a reduced need for maintenance. These two aspects of a structure can be planned for in the original design, and they will be evaluated qualitatively in the Miami Marine Stadium. Finally, a discussion of the significance of the stadium to the city of Miami will detail how the structure interacts with its urban context. This section fleshes out the life of the stadium in the past, present, and future, as each represents a distinct phase in the life of the structure.

The research contained in this section relies on both primary and secondary sources associated with the stadium. As part of the research process, a trip was taken to Miami in February 2012 to document the history and current condition of the stadium as well as its urban context. This trip coincided with an event held to honor Jack Meyer, the engineer responsible for implementing the design of the stadium along with its architect, Hilario Candela. The event was hosted by the Friends of Miami Marine Stadium, a non-profit organization which advocates for the restoration of the stadium. Meyer, Candela, the Mayor of Miami, the Friends of Miami

Figure 5.2: A photo of the author with stadium engineer Jack Meyer, showing the skyline of Miami in the background.

Marine Stadium Board of Directors, and many other individuals associated with the stadium were present at the event, providing ample opportunities for personal conversations concerning the stadium's design and history. In addition to these conversations, many hours were spent at the stadium with Meyer himself discussing the stadium's design and construction and taking pictures and video. The Friends of Miami Marine Stadium also operate a website containing links to a wealth of media concerning the stadium. Given these sources, an exhaustive account of the stadium's history is possible, but this chapter seeks only to highlight the most important information about the concrete thin shell's design, construction, significance, and context.

5.2. Design

The overall form of the stadium was first conceptualized in the mind of the architect Hilario Candela, a Cuban-born designer who was 26 years old when he was given the assignment. He had been given the assignment for a grandstand situated beside a dredged basin after a need had arisen for a viewing area and racecourse for speed boat racing in Miami's Biscayne Bay. According to Jack Meyer, the eventual engineer of the stadium and a native to Miami, some of the pressure for the racecourse came from the Orange Bowl Committee, which oversaw the events associated with the famous college football game (Meyer, 2012). At that time, the Orange Bowl was still played in the original Burdine Stadium in downtown Miami, and the Orange Bowl festivities traditionally included boat racing in the bays surrounding the city. As the waters became polluted with trash and other substances, it began to negatively affect the boat engines, and with time racers began to avoid the Miami circuit. Thus, the organizers of the Orange Bowl desired a new artificially created course for boat racing that would continue to attract top racers to the event (Meyer, 2012). The city of Miami agreed, and in 1962 the Chicago planning firm of Ralph H. Burke submitted a master plan for a recreational landscape on the Virginia Key, situated less than two miles off the coast of Miami, in between Key Biscayne and Miami Beach (Adriaenssens *et. al.*, 2012). Once realized this plan would be the world's finest and most unique speedboat course. A dredged aquatic basin for a similar purpose already existed in Long Beach, California, and a waterside grandstand was built at Jones Beach Theater in Wantagh, New York ten years earlier, but the combination of the two has never been realized elsewhere (Adriaenssens *et. al.*, 2012). The site would be closed on three sides, but offer views of the Miami skyline on the open fourth side, to the left of viewers in the grandstand (See Figure 5.3).

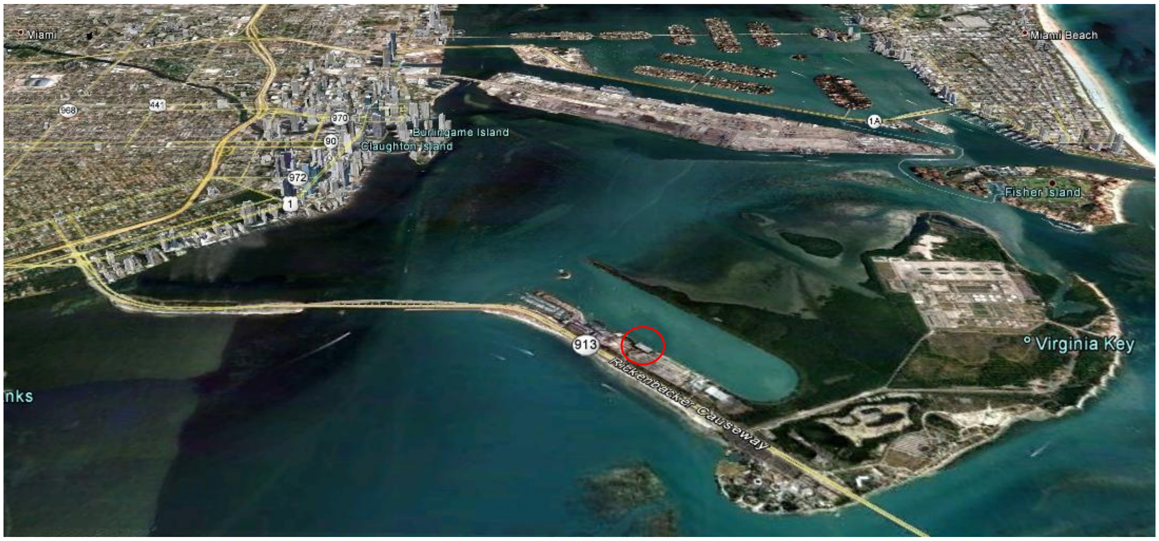


Figure 5.3: Oblique view of Virginia Key, the location of the Miami Marine Stadium. The stadium sits on the edge of a three-sided aquatic basin, with views of downtown Miami to the left. Image courtesy of *Google Maps.*

The master plan called for a typical baseball field grandstand with a metal truss roof, which would be an expected architectural expression for the stadium. However, once Candela was given the project by the firm of Pancoast, Ferendino, Skeels and Burnham, it was clear that he would not accept such a design and instead would form his own out of concrete. One of the main influences on Candela's design was his interest in clearly expressing the structural behavior of a building through its form. In a 2011 interview, Candela stated that he viewed structure "not as a tool to support a building, but as a visible architectonic expression" (Candela, 2011). In addition, Candela had a fascination with engineered concrete, which began with his exposure to the work of Nervi, Torroja, and Félix Candela during his time as a student at Georgia Tech. Candela himself also had experience in thin shells, having worked with Max Borges Jr., Raul Alvarez, and other members of Cuba's shell-building architectural avant-garde before leaving his home country to work in Florida. These general influences and experiences gave way to a more direct design inspiration when in 1962, as Candela was beginning to design the stadium, he happened to fly into Dulles Airport in Washington, D.C.. The way in which the airport roof sat on top of the columns like a canopy and the manner in which the sloped, smoothly plastic concrete columns carried its weight down to the ground inspired Candela to do the same with his stadium (Candela, 2011).

Figure 5.4: Dulles Airport Terminal in Washington, D.C., with its slanted concrete columns. Photo courtesy of *Wikipedia Commons*.

Candela also took other factors into account besides the visual effects of the stadium, since he viewed the use of concrete as "both an aesthetic and a civic statement that allowed for economic functionality while maximizing the 'softness and strength'" of the material (Candela, 2011). At Jack Meyer's event, Candela stated there were only two aspects of the design for which he would not compromise (Candela, 2012). First, the stadium had to be beautiful. Second, it had to come under budget. During the preliminary design stages, Candela had been threatened by city officials wary of the innovative concrete form that if he could not produce a concrete stadium under budget, the city would force him to design a more conservative steel grandstand on his own dime (Candela, 2012). Keeping these constraints in mind, Candela nevertheless used his own experience in shell building and inspiration from his predecessors to draw an elegantly curved, hypar umbrella roof supported by slanted, sculptural columns. When he passed these sketches off to prospective engineers for the stadium, he was confident it could be constructed efficiently and economically.

When the initial concept drawings were given to the firm of Norman Dignum Engineers, every engineer except for Jack Meyer refused to take the immensely difficult project (Meyer, 2012). Even Meyer, despite his willingness to accept a challenge, had plenty of initial misgivings about the design. According to Meyer, “Hilario was a very skilled artist who wanted his roof to float on top of basically nothing, and he wanted holes in the seating area around the columns big enough to throw a cow through” (Meyer, 2012). Initially thinking the dramatically thin curves of the roof and the shape of the columns were not possible, Meyer countered with a much flatter, bulkier, and more conservative design for the roof. Candela refused to yield on the overall shape, however, a decision which gave Meyer “plenty of headaches and sleepless nights”, but as the engineer now concedes, resulted in a much more beautiful stadium. Although there were plenty of meetings between architect and engineer, Candela primarily communicated through sketches and by vetoing or accepting engineering solutions that changed the visual appearance of the stadium. As such, it was up to Meyer and other members of his firm to develop the actual dimensions of the design.

At the time, Meyer had a modest amount of experience in designing with concrete, especially in such complex arrangements. Until his design of the stadium, he had worked on mostly schools and a number of projects for the Navy, but not many major public structures using concrete as the medium (Meyer, 2012). It is worth noting that a few years before the stadium, Meyer designed the foundation and some of the superstructure for a complexly curved, roughly cylindrical water tower. Although he had not been responsible for designing the formwork for the complicated, curved structure, the workers’ ability to create such a complicated network of curves out of initially straight boards impressed him and set a precedent in his mind for generating curved formwork (Meyer, 2012). Meyer’s firm had also recently completed the structural work for a number of large folded-plate concrete roofs for bowling alleys. Using his knowledge of both curved, poured in place concrete structures as well as folded-plate roofs, he decided to design a structural system using elements from both which would ultimately converge to closely resemble the shape of Candela’s design.

Although there was collaboration between Candela and the engineering firm, Meyer stated that the majority of his design time was spent in-house trying to establish the shape of the structure and then performing iterative calculations. Meyer praised the work of his firm’s draftsmen, who could efficiently use tables to follow the hyperbolic geometry of the roof and helped him establish the dimensions of the design on paper. Using American Concrete Institute (ACI) code, Meyer calculated the shells essentially as a folded plate, adding structural elements where he saw fit as long as they were in line with Candela’s aesthetic sense. Since the stadium was such a complicated structure, there were admittedly times when Meyer added elements without complete knowledge of how they were going to behave. For example, when told after the fact that Finite Element models show the stiffener could possibly be removed without adversely affecting structural performance, Meyer replied: “I am not surprised by it, but I knew someone back then who used a lot of post-tensioned stiffeners and recommended one for this project, and I sure wasn’t going to leave it out” (Meyer, 2012). The shape of the stadium also forced Meyer to add unconventional design aspects that he did not wholeheartedly trust until they worked, such as sharply bending the rebar to follow top peak of the stadium rather than tying two rebars together, which was difficult due to the lack of cover in the thin shell. However, Meyer also made design decisions that were fully understood and highly effective, such as using lightweight concrete in the cantilever to balance out the weight of the cantilever with the weight of the backspan on the central column. The end product was a design that closely followed the geometry proposed by Candela, but was based on sound engineering principles and intuition.

5.3. Construction

Both the construction procedure design and its actual execution are among the most important factors in determining the cost of the stadium. Although the architect and engineer were involved, most of the actual procedure was designed by the subcontractors themselves. At the time the stadium was built, the economic situation in Miami was such that there was little work to go around for builders (Adriaenssens *et. al.*, 2012). As a result, the stadium was able to attract the very best subcontractors for each aspect of the stadium, since they were all desperate for work. The main contractor, a small local construction company called Mailman Construction, was selected through the normal bidding process, and was able to partner with the most advanced steel and concrete workers to submit the lowest cost (Adriaenssens *et. al.*, 2012). Many subcontractors were excited to be a part of such a difficult project. As Candela notes years later, one of the distinct ingredients in the construction of the stadium was pride: "Each person associated with the stadium, from me to Jack to the workers setting the steel and pouring the concrete, took pride in their work" (Candela, 2012). It is vital for the success of any shell that it be constructed precisely, and the Miami Marine Stadium was not an exception to this rule.

Since the stadium design was unlike most work they had ever completed, the subcontractors thought deeply about how to build the stadium. Meyer recalls one meeting in which the steelworkers called him to their office to discuss the reinforcing plan for the stadium. The steelworkers had a fairly large office, and when Meyer arrived they had a full scale mock-up of an entire umbrella spread out over their floor. They meticulously went over the whole system with Meyer, calling attention to certain spots they thought would be problematic while working with the engineer to finalize the design (Meyer, 2012). The main contractor also built a complete model of the stadium at around 1/20 scale to plan for an economic construction procedure. After building this model, Mailman Construction decided to pour the wet concrete in place over plywood formwork, which would be supported by steel scaffolding (Adriaenssens *et. al.*, 2012).

Mailman had skilled carpenters construct two molds, which were reused three times to create the eight umbrellas. In addition to the stadium design being tessellated to save money, Candela and Meyer understood that the geometry of a hypar allowed for the formwork to be generated from straight lines, and the use of thin straight boards contributed to low construction costs. Meyer also specified that galvanized steel be used in construction, which protected against corrosion from the saltwater surrounding the stadium and the seepage of water through the highly permeable lightweight concrete in the main cantilever section (Simpson *et. al.,* 1993).

During the actual construction, Meyer regularly monitored the site, although there were also two other inspectors from the city of Miami. In general, Meyer was pleased with the thoroughness and quality of work in the stadium. He did not have to make many corrections because in his view, "the structure was radical enough that it scared the workers, enough that they were careful to do things the right way so it would not fall down" (Meyer, 2012). The stands of the stadium were built first, with the roof added soon after. The scaffolding and forms were erected for the roof two at a time, at which point the steel reinforcing was placed on top of

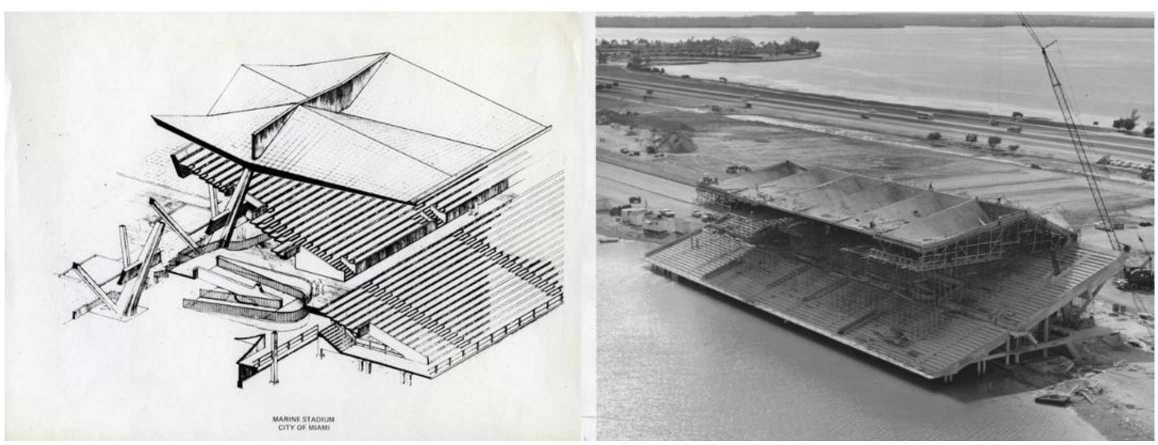


Figure 5.5: Section drawing of stadium on the left, showing various structural elements. A photo taken during construction on the right, showing the reused formwork. Images courtesy of *Friends of Miami Marine Stadium*.

the forms. Once the forms and steel were in position, workers used wheelbarrows to carry concrete over the forms and pour it in place (Adriaenssens *et. al.,* 2012). In some areas of the roof, workers attached small boards to the formwork, and marks left from these boards can still be seen on the stadium today (See Figure 5.6). The most dramatic moment in the construction of the stadium was the removal of the first formwork. Meyer calculated that the deflections may be as high as a foot and a half (45.7 cm) at the tip of the cantilever, although even three feet would be

Figure 5.6: Marks left by boards placed on the stadium by workers for footholds, still visible today.

acceptable for the structural integrity of the roof. To his satisfaction, when the formwork was removed the tip of the cantilever only slumped about 6 inches, or 15 cm (Meyer, 2012). These deflections were effectively reduced by the curved shape of the structure, as well as steel weld tabs between umbrellas, which were covered in concrete grout and prevented relative translation between umbrellas (Adriaenssens *et. al.*, 2012). Overall, the designers created an innovative, cost-efficient construction procedure that conserved financial and material resources while building a strong, well-engineered structure.

5.4. Performance and Sustainability

The Miami Marine Stadium has been praised by many members of the construction and concrete industries for its durability and sustainability despite its lack of maintenance over the years. Although the stadium was well kept over three decades while playing host to many races, concerts, and other Miami events, the turning point in the condition of the stadium occurred in 1992 in the aftermath of Hurricane Andrew. Thinking that the stadium had been severely damaged, the city of Miami closed the stadium and hired the structural engineering firm of Simpson, Gumpertz and Heger to assess its condition. The firm found that although the regions of the stadium located in the "splash zone" had deteriorated severely, the hypar roof was in good structural condition (Simpson *et. al.*, 1993). The main issue with the roof was the cracking pattern found throughout the thin shell region. The report concluded that "the analysis and

Figure 5.7: Cracking pattern on the shell roof, accentuated by rain running through the cracks to the roof's underside. The view on the right is zoomed out, showing a larger view of the pattern.

design procedures used for the design of the Miami Marine Stadium achieved a safe design but one that was prone to cracking particularly at the thin lightly reinforced front hypar shell. The existing cracks are not are not cause for concern regarding the structural integrity of the roof, however" (Simpson *et. al.*, 1993). Although water was flowing through these cracks in the roof, the reinforcing was not corroding at a dangerous rate because the design used galvanized steel. Thus, the stadium remained strong enough to withstand the high winds of a category four hurricane, coming out in fairly good condition on the other side of the storm.

Despite the structural health of the roof, the stadium nonetheless required rehabilitation in order to be reopened, and the city of Miami was unable or unwilling to provide this maintenance. The city decided to close the stadium, at which point it became a magnet for graffiti artists, skateboarders, and other Miamians looking for a place to pass the time. Without being trimmed, trees and other vegetation grew up around the exterior of the stadium, obscuring its view from Rickenbacker Causeway, the road running behind it on the way to Key Biscayne (see Figure 5.8). At present, nearly every surface imaginable is covered by spray-paint, from the seats, floors, and walls to the roof above (see Figures 5.9 - 5.11). The underside of the shells remain fairly untouched by paint, but graffiti artists were even able to 'tag' the edge beam and stiffener of the roof by accessing the top of the shells by way of a precarious plank from the hanging press box. Nevertheless, through nearly two decades of complete neglect and vandalism (or perhaps because of the vandalism, depending on perspective), the stadium has maintained its beauty and structural soundness, enough to keep the attention of Miamians who remember the stadium in its heyday.

Figure 5.8: A current view of the stadium, obscured by untrimmed vegetation.

Figure 5.9: View of broken stadium seats (top) and graffiti covering the supports (bottom).

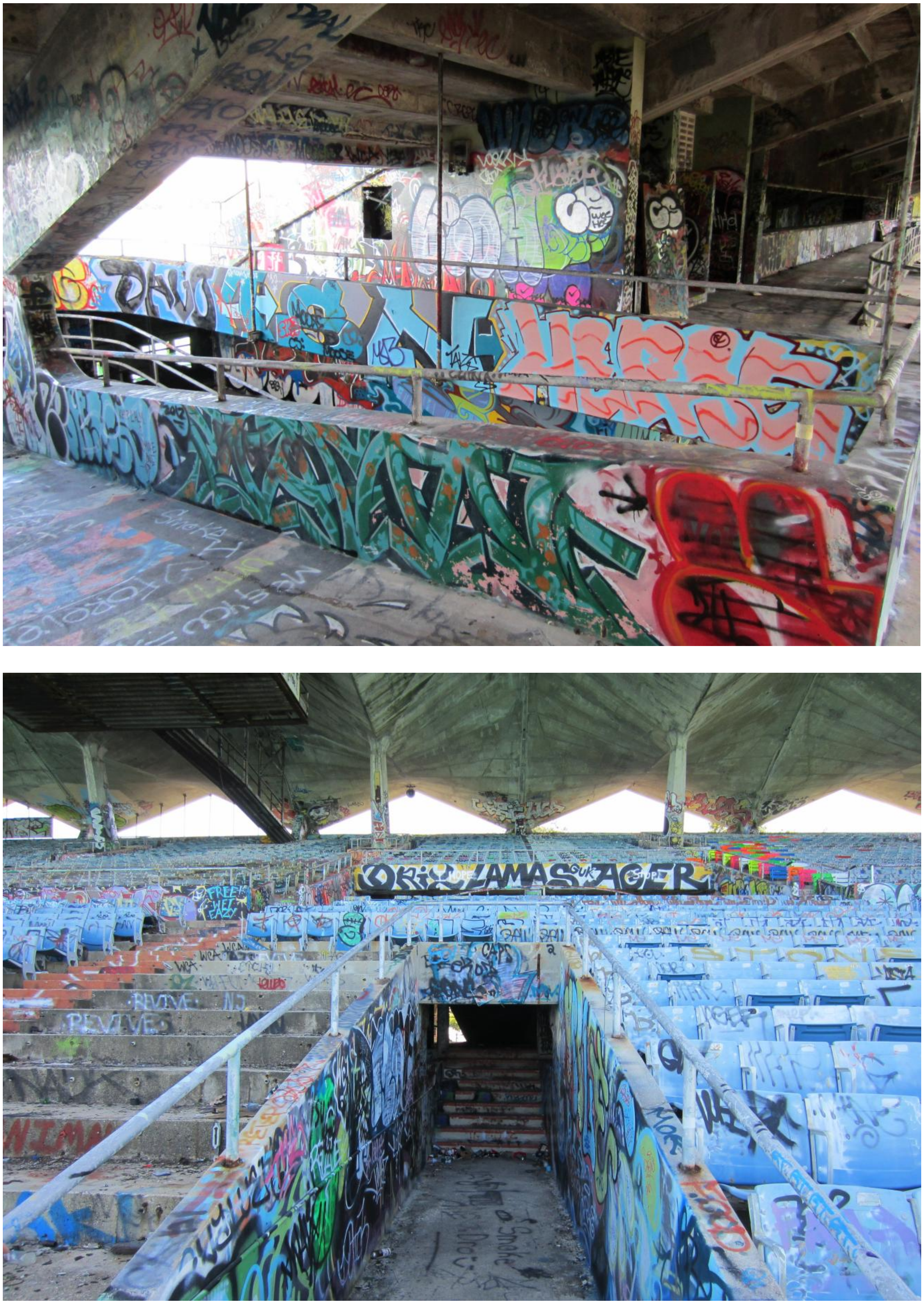


Figure 5.10: Graffiti in the back walkway (top) and in a seating entrance (bottom).

Figure 5.11: Graffiti covering a ramp (top), and the pressbox hanger (bottom).

In 2009, SGH was hired to perform a reassessment of the condition of the stadium to estimate the cost of repairs and identify methods for protecting the structure to increase its usable life. SGH concluded that although the deterioration had continued since their report following Hurricane Andrew, it remained similar in nature and had not accelerated alarmingly (Brainerd *et. al.*, 2011). Portions of the structure located in the "splash zone", which are not positioned completely underwater but are contacted by alternating saltwater spray and oxygen, showed moderate to severe deterioration. The seating area, support columns, beams, and parapet walls exhibited moderate deterioration with localized spalls or delaminations as well (Brainerd *et. al.* 2011). As was true in 1993, the roof was still in excellent structural condition, suffering only from cracking, which SGH recommended could be sealed through the injection of epoxy, adding tensile strength to the shell (Simpson *et. al.*, 1993). A number of holes in the concrete also exist, but these are for the most part small and insignificant.

Given the harsh marine environment of the stadium, its designers have been praised for creating a highly durable and sustainable structure. Meyer's use of galvanized steel has been singled out for being visionary and lauded by concrete experts, since its use was unusual at the time. According to the International Concrete Repair Institute, the Miami Marine Stadium is in better condition than a number of other historical stadiums such as the Rose Bowl in Pasadena, despite no repairs or maintenance in the last twenty years (Meneely, 2012). In their final report, SGH concluded that "the repair and rehabilitation of Miami Marine Stadium for safe public use is technically feasible and practical" (Brainerd *et. al*., 2011). Rehabilitation would include corrosion mitigation measures and other direct repairs, but is not so extensive as to completely reinvent the original structure. According to SGH, "all of these repairs and preventative measures can be achieved in a way that preserves and maintains the significant architectural and historic character of this modernist icon" (Brainerd *et. al*., 2011). Thus, the Miami Marine Stadium serves as an excellent example for how thin shell concrete structures can be built to last even with minimal maintenance, an important component of sustainability as it relates to structural engineering.

5.5. Importance to the City of Miami

To many citizens, the Miami Marine Stadium is a cultural landmark of the city akin to the statue of Liberty in New York, the St. Louis Gateway arch, and the Seattle Space Needle (*Friends of Miami Marine Stadium*). The stadium is currently the focus of a powerful restoration effort, a testament to its significance as part of the city's identity. The Friends of Miami Marine Stadium, an all-volunteer organization, was founded in 2008 to advocate for the saving of the stadium, as well as plan its eventual reintegration into the urban fabric of the city. Since that year, the stadium has been named to the National Trust for Historic Preservations "11 Most Endangered Historic Places", generating a considerable amount of national attention. The restoration of the stadium has also gained support from Miami mayor Tomás Regalado, singer Jimmy Buffett, the University of Miami School of Architecture, and the World Monument Fund (*Friends of Miami Marine Stadium*). The many organizations and people who are part of the restoration effort are general evidence of the stadium's importance, but aspects of the stadium must be explored directly to understand the root of the admiration Miami has for its concrete hypar structure.

A discussion of the significance of the Miami Marine Stadium within its urban context can begin with anecdotes from the many people who attended an event there over the years. These stories range from simply nostalgic to displaying a deep understanding of the stadium as a civic and cultural icon of Miami. At the time of writing, Don Worth, co-founder of the Friends of Miami Marine Stadium, is collecting many of these stories with the intent of publishing them in a series called *If Seats Could Talk*. Worth summarizes the sentiment behind many of these statements, exclaiming of the stadium: "this is where Sammy Davis Jr. hugged Richard Nixon. It's where Pete Townsend of the Who lit his guitar on fire, threw it in the water and a dozen people at the concert jumped in after it. There are so many memories" (Sasser, 2010). Many of the treasured memories indeed come from concerts at the venue, which were made possible by a floating stage situated out in the basin. For jazz, rock, blues, and many other types of concerts, residents of Miami would fill the stadium and the water in front of it. It was not uncommon for

Figure 5.12: A concert at the stadium held on a floating stage. Photo courtesy of *Friends of Miami Marine Stadium*.

people to anchor their boats in the water adjacent to the stage or even to float on a surfboard for the duration of the concert. According to Lulu Hart, a longtime Miami resident, "concerts under the stars with the Miami skyline as a backdrop were absolutely magical" (*Friends of Miami Marine Stadium*). During the research trip as part of this thesis, it was nearly impossible to meet a Miamian older than the millennial generation who did not have a fond memory of attending a concert at the stadium, including taxi drivers and hotel staff who had no association with the restoration effort.

Much of the nostalgia associated with the stadium also comes from memories about boat racing, an activity deeply engrained in the Miami culture and the purpose for which the stadium was originally built. Michael Lemieux, another Miami resident, wrote: “imagine the feeling; walking through the bleachers, filled with adrenaline and anticipation, going to see the fastest boats in the world! I heard a noise…then a red blur, then the whine of 2500-plus hp turbine engine, as a hydroplane whizzed past me at over 200 mph. I loved it and will never forget”

Figure 5.13: A boat race at the stadium. Photo courtesy of the Friends of Miami Marine Stadium

(*Friends of Miami Marine Stadium*). A man named Jim McKeon who watched races as a child is hopeful for the reopening of the stadium, writing that “to see the unlimited hydroplanes roaring again on the course would be a dream come true”. McKeon also noted that the history of the stadium is not gone unless the city chooses to collectively forget. Boat racing is a vital part of Miami’s culture, which is rather unusual for such a large city. Nevertheless, the memories of races at Miami Marine Stadium solidify the importance of the stadium to the city itself.

There is also a strong generational component to the stadium nostalgia, with older citizens bemoaning the experiences robbed from their children due to lack of maintenance of the stadium. Many citizens have fond memories of attending events with their parents, and have expressed interest in taking their children to the stadium once it is restored. In her comments about concerts at the stadium, Hart also wrote: “I’ve told my son all about the Marine Stadium, and I hope to one day take him to a concert there that will blow his jaded teenage mind” (*Friends of Miami Marine Stadium*). No less than the mayor of the city himself expressed a similar sentiment. Speaking at Jack Meyer’s event, Mayor Regalado said that “although I have many fond memories of the stadium, unfortunately my children were not able to share in these events. It is my hope that with this restoration effort, my grandchildren will be able to enjoy all the stadium has to offer” (Regalado, 2012).

These memories speak to the city’s past, as well as its future. Although South Beach may be a glittering tourist attraction and nightlife destination, to the south, older residents of the mainland and keys bordering Biscayne Bay recall the time before the giant real estate booms occurred in South Florida. This Miami was built by families with long ties to the region and an affinity for the older, marine-centered way of life. Many people associated with the stadium restoration proudly declare themselves as third, fourth or fifth generation Miamians. An example of this long term association with the city is the stadium engineer himself, Jack Meyer. Meyer recalled once rowing a boat out to Key Biscayne “when the whole place was just a swamp”, and getting it stuck (Meyer, 2012). Unable to free his boat, Meyer had to swim back across the bay to reach his home right past Virginia Key, the eventual site of the stadium (Meyer, 2012). Now, much of the area is full of high-rises, connected to the mainland by a massive toll causeway. The stadium itself, as well as the city, must confront this change in the city’s makeup as it pursues relevance in the future.

Although the romantic attachment of citizens to the stadium can substantially enhance an understanding of its significance, the stadium's relationship with the city is more complex than memories of the events held there. The stadium is architecturally significant and culturally representative, as was planned by the architect. In an article published by *Modern Magazine*, Rebecca Kleinman explains the architectural context in the stadium. "Miami Beach has always hogged the limelight in the Florida metropolis, when it comes to architecture" (Kleinman, 2010). In the rest of Miami, architects "did not forge a cohesive design identity", focusing primarily on residential architecture instead of large public works (Kleinman, 2010). The Marine Stadium is one of the few large public structures that pushed the architectural envelope and achieved success. In the words of critic Tom Austin: "from the beginning it [the stadium roof] looked like a fighter jet about to take flight, a modernist gem as perfectly crystallized in its architectural balance as a hawk's wing" (Austin, 2011). Thus, the stadium stands nearly alone as a tribute to Miami's affinity for Mid-Century Modernism. Modernist design is an important component of Miami's architectural heritage, and one that should be preserved in the few large examples left in Miami as a historical anchor point for the city.

The stadium is also architecturally important because it embodied the spirit of Miami for many years, and the architect hopes it will be able to do so again. When envisioning the eventual use of his stadium Candela noted, "Miami was just growing and it [the stadium] coincided with a freedom of imagination that, from a cultural point of view, our society had…people realized that Miami had a destiny that was unique, that was related to tourism and to the water" (Melnick, 2012). Both are of course true: Miami has long embraced tourism, using the prospects of luxury accommodations, restaurants and clubs, pristine beaches, and entertainment events to lure in visitors (*City of Miami*, 2011). Miami is always putting on a show, even in the most literal sense of adding colored lighting into its major buildings' facades at night. In this context of showmanship, according to Kleinman, the stadium was always equal to the spectacle (Kleinman, 2010). The stadium's architectural expression embraced this aspect of the city to such a deep level that it was able to integrate into the spectacle itself: wrote Austin, "to take in the swing and sass of Ray Charles, with the stadium bathed in moonlight, was to realize the romantic possibilities of Miami" (Austin, 2011). As the city has grown, the stadium's size makes it unlikely that it can serve as the center for this spectacle moving forward. However, the stadium will certainly be able to take part in Miami's continuous show, while at the same time specializing to a more precise significance. Candela hopes that with the restoration of the stadium and construction of a nearby park and marine center, Virginia Key "could be the headquarters for boating and boaters…not the corporate headquarters – the soul headquarters" (Melnick, 2012).

In light of Candela's sentiment, restoration efforts have moved past the advocacy stage and are currently raising the money in an attempt to develop the whole area as an events venue with an economically sustainable business model. The turning point came in July of 2010, when the City Commission of Miami approved a new Master Plan for development of Virginia Key that included not only saving the stadium but made it the centerpiece of the entire plan (*Friends of Miami Marine Stadium*). The stadium's restoration is now a priority of Miami's mayor, and the Friends of Miami Marine Stadium have established relationships with local organizations willing to contribute to the restoration or eventual management of the stadium. If thought is put into Miami's cultural, social, and economic present and future, it is likely that the stadium will be successfully reintegrated into the city's urban fabric.

5.6. Conclusion

In the case of the Miami Marine Stadium, a series of compromises between two exceptional yet different designers created a beautiful, sustainable, economically viable thin shell structure. The architect Candela was responsible for the overall hyperbolic form of the stadium as well as providing the vision for what the stadium could eventually become, a modernist architectural marvel that reflects the vibrant spirit of Miami and considers the marine history of the city. This vision came from a deep understanding of the cultural aspects of Miami as well as knowledge of structural precedents set by thin shell masters of the past. Spurred on by both historical examples and an innovative drive, the stadium design called for a beautiful form, versatility of use, comfort for patrons, and a low cost. The engineer Meyer was able to take this ambitious vision and turn it into reality by using sound engineering judgment and a creative approach to the structural challenges posed by the stadium's shape. Despite a lack of tools available to analyze the structure, Meyer was precise and pragmatic, yet daring enough to propose a cantilever system that had never been done before. Together, these designers illustrate the viability of sustainable concrete thin shells in their own context and can inspire future designers to produce similarly imaginative and successful structures.

6. Structural Study of the Miami Marine Stadium

6.1. Introduction

The purpose of this chapter is to provide an in-depth analysis of an existing structure, which will lead to a better understanding of how the chosen shape of a concrete shell can beneficially influence its structural performance. This thesis uses the Miami Marine Stadium roof (See Figure 6.1) as a case study for a hyperbolic paraboloid (hypar) structure, since this roof exhibits a favorable relationship between form and forces. The stadium roof is essentially made up of 8 hypar umbrellas, with each one standing 20.2 m tall and having a width of 12.4 m. Each umbrella contains a 20.2 m cantilever running out towards the water, with a 10.5 m backspan that sits on both the main umbrella supports and additional columns slanting out towards the back of the stadium. Including edge stiffeners, the overall footprint of the stadium is 99.5 m wide by 30.7 m deep (Pancoast *et. al.* Consulting Architects, 1964). The overall visual effect from the side is a two-support cantilever with a free, straight edge. The front of the stadium shows the underside of the hypar shells meeting at this straight edge, while the back contains ridged, wave-like arches running towards the visual plane of the stiffener.

Figure 6.1: Views of the Miami Marine Stadium. Photos courtesy of *Friends of Miami Marine Stadium.*

The Miami Marine roof has a more complex shape than a simple hypar, borrowing elements from a folded plate system as well. With no advanced computational analysis techniques available at the time for its complex curved shapes, the engineer of the stadium, Jack Meyer, designed and analyzed the roof as eight connected varying depth cantilevering beams using a series of conservative analytical calculations based on flexure, slope, and displacement formulas (Adriaenssens *et. al.,* 2012). Unfortunately, record of these calculations no longer exist and cannot be referenced in this thesis. Instead, the effects of certain design elements are tested in a comparative analysis between a number of different models in order to understand the structural performance of the stadium as it was built as well as ways in which the stadium design could be simplified or improved. It was hypothesized that if the folded plate system elements were removed from the stadium, the structural performance would not be adversely affected despite the considerable reduction in material. If this is true, then the shape of the stadium is well suited to pure hypar shell behavior. If the stadium roof instead fails structurally without the folded plate elements, then this study proves that the stiffening groins and diaphragm were necessary, and the hybrid structural system is the best solution to the problem.

In order to complete this analysis, numerical modeling is employed through computer-driven Finite Element Analysis (FEA). The FEA software package LUSAS, which consists of both a sophisticated modeler and a solver, was used to carry out the structural study. LUSAS contains an extensive library of structural elements, meshing options, materials, and loading methods, and selections from each of these categories were used in the development of the stadium roof models. This chapter begins with a description of the four different models used for comparative analysis. A description of the modeling methodology comes next, followed by a detailed explanation of the different characteristics of the LUSAS models and modes for solving the finite element problem. Finally, a discussion of results is presented first in terms of each individual model, and then combined and contextualized to synthesize the broader implications of the structure's form and behavior.

6.2. Model Presentation

This chapter presents a finite element analysis of four separate models, which will be referred to as FEM-1 through FEM-4. The first model (FEM-1) represents the stadium as it was constructed by using variable shell thicknesses for elements near the groins of the roof (See Figure 6.2). In this first model, the thickness of the main cantilever section varies from 7.62 cm in its thinnest region to over 50 cm in the groins. Although it is still being modeled as a matrix of thin shell elements, the thickened groins are a common characteristic of folded plate design and should cause the structure to behave in a plate or beam manner, which is consistent with typical folded plate systems.

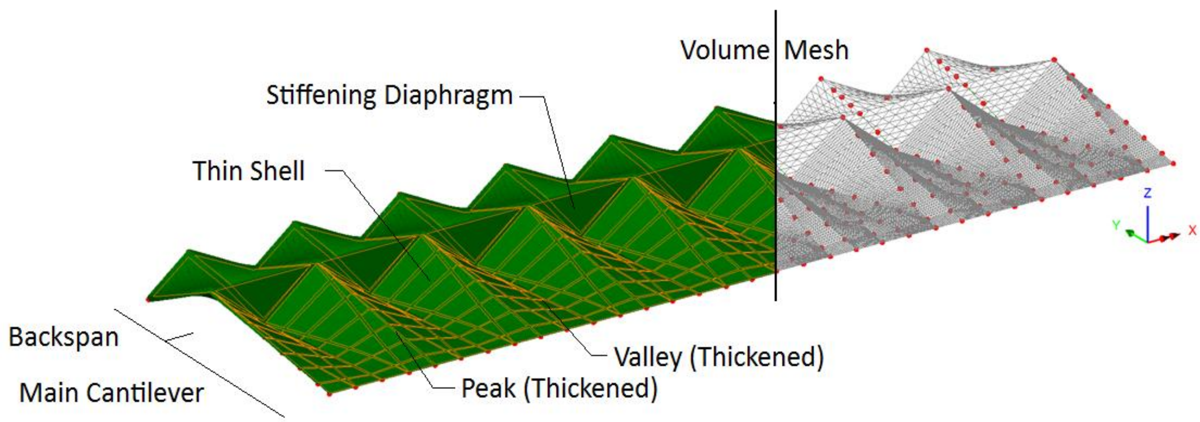


Figure 6.2: Various structural regions of stadium roof.

The second model (FEM-2) uses the same geometry but re-conceptualizes the stadium roof as a series of hypar shells with a constant thickness of 7.62 cm throughout the front cantilevered section. This model drastically reduces the amount of concrete in the structure, and a comparison between the two models provides insight into how necessary the existing thickened groins are in order for the roof to be functional. The third (FEM-3) and fourth (FEM-4) models correspond to the first and second, respectively, except that they do not contain the stiffening diaphragm. Diaphragms such as the one in the Miami Marine Stadium contribute substantially to the performance of folded plate structures (Ketchum, 1997), but are unnecessary in hypar-shaped thin shells. It was hypothesized by master builder Félix Candela that the form of this stadium dictates hypar behavior and that the diaphragm is not necessary in the design (Adriaenssens with H. Candela, 2011) . This thesis tests Candela's hypothesis by removing the stiffening diaphragm and seeing whether the performance of FEM-1 or FEM-2 is more affected by the lack of a stiffener. In either case, the magnitude of the differences in stresses and deflections with and without the diaphragm should show whether the roof's form tends to behave more like a folded plate or more like a hypar. A concise, detailed description of each of the four models is given in Table 6.1and Figure 6.4:

Table 6.1: List of Different FEA Models

Model Name	Cantilever Shell Thickness	Existence of Diaphragm	Meshing
FEM-1	Variable from 7.62 to 58.42 cm	Yes	Triangular shell elements quadratically interpolated with 16 local divisions
FEM-2	Fixed at 7.62 cm	Yes	
FEM-3	Variable from 7.62 to 58.42 cm	No	
FEM-4	Fixed at 7.62 cm	No	

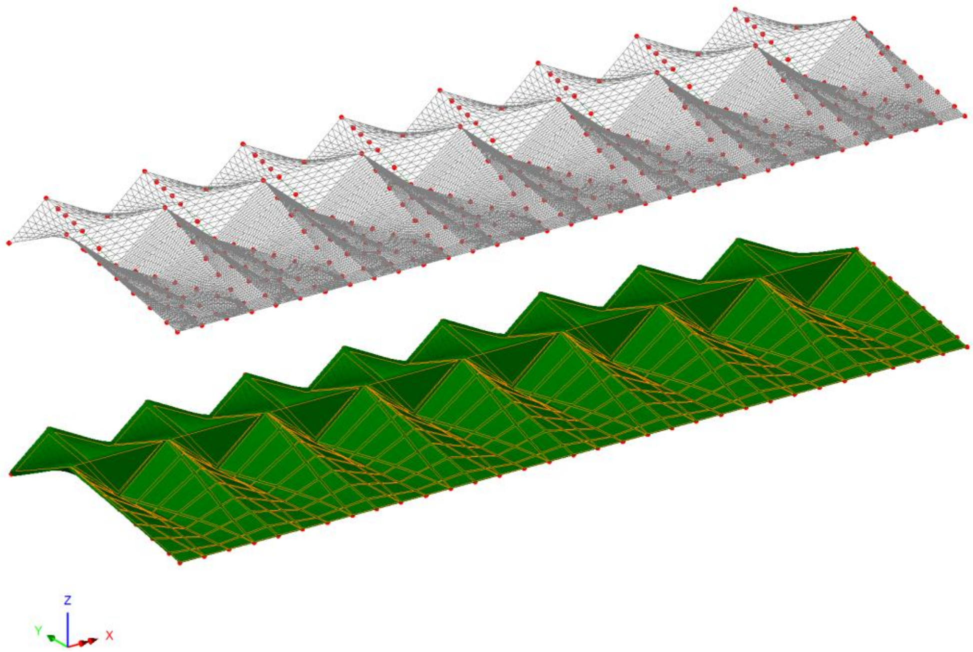


Figure 6.3: FEM-1 Meshing and Visualization (FEM-3 is the same except without the diaphragm).

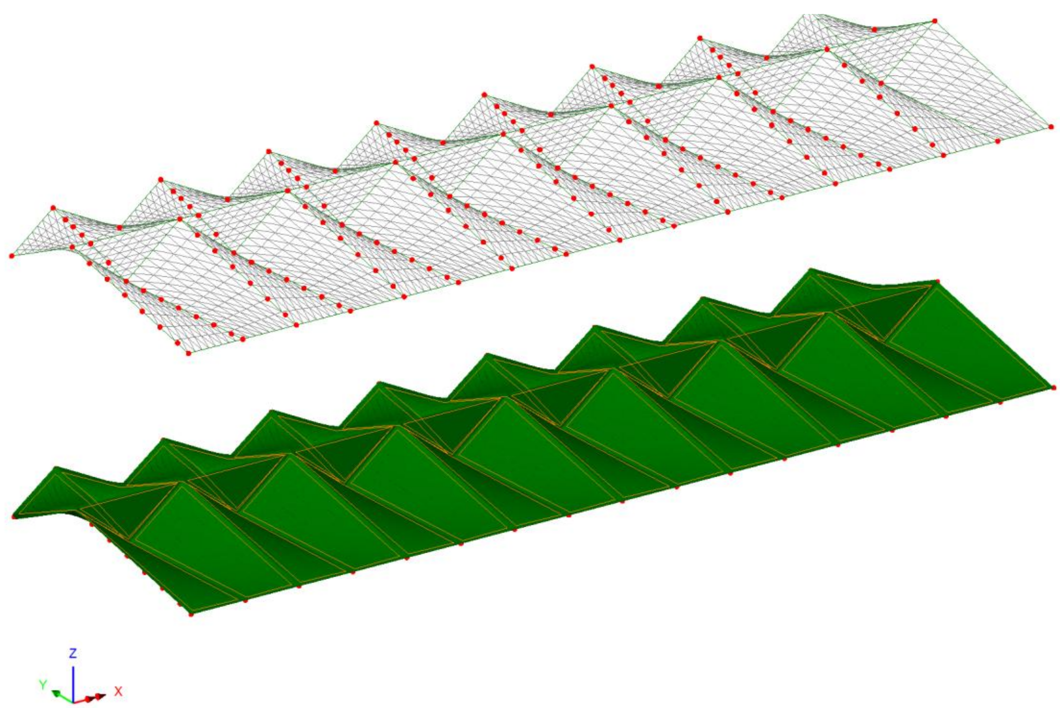


Figure 6.4: FEM-2 Meshing and Visualization (FEM-4 is the same except without the diaphragm).

6.3. Model Methodology

The form of the Miami Marine Stadium roof is a complex shape that does not follow a specific surface equation. Therefore, the most effective way to model it was to study the construction drawings and transfer its geometry to the LUSAS modeler point by point. After looking at the drawings, a number of key points in the geometry of the roof, such as the corner of an umbrella or a point at which the thickness of the shell changes discretely, were assigned absolute coordinates in a three dimensional LUSAS model. It was then possible to connect these points into lines and form the actual surfaces of the roof from these bounding lines. Since each of the eight umbrellas of the stadium are identical except for features that were removed in the design simplification stage, it was only necessary to trace the geometry for one umbrella, since it could be arrayed in a linear, side by side fashion. Views of the skeleton geometry of the stadium roof are given in Figure 6.5 and Figure 6.6.

Figure 6.5: Isometric views of the basic geometry.

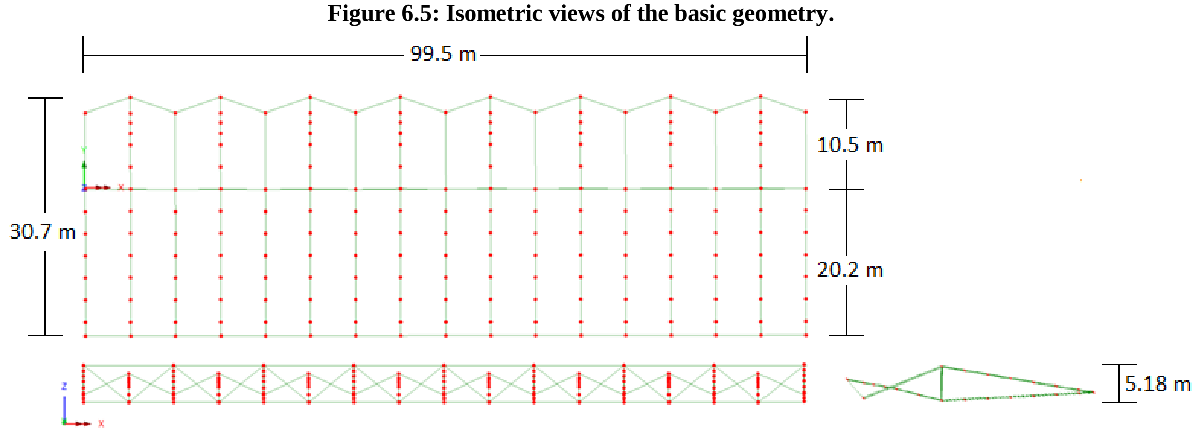


Figure 6.6: Plan and elevation dimensions of the basic geometry.

After completing the main geometry of the stadium, attributes were added to each surface. These attributes are stored as properties of the surfaces and are taken into consideration by the solver. The way in which each attribute affects the final solution will be discussed in detail in each respective section, but a broad overview of the process of assigning attributes is given here. The first attribute added to the geometry was meshing, which is generated automatically from given inputs such as element shape, interpolation order, number of local divisions, and an allowance for transfer patterns and irregular mesh in oddly shaped surfaces. The meshing was visualized by LUSAS so that it could be inspected for irregularities. Since the type and size of meshing has a substantial effect on the final results and there are complications associated with certain meshing choices, simplified experimentation was done to determine the most feasible meshing for the model. For example, as the local divisions got higher and higher, the solver would take exponentially longer to finish a problem. Thus, a number of local divisions was selected based both on suggestions in the LUSAS manual and also by limiting the number to give problems that could be reasonably computed. In addition, when the stadium roof was broken into smaller, complex surfaces to model the variable areas of the groin vaults, extensive irregular meshing and more local divisions were sometimes needed to complete the surface. Irregular meshing lead to asymmetry between certain areas of the roof and in some cases allowed concentrations of very small elements to occur (See Figure 6.7). When this was discovered, the geometry was smoothed at a few of the joints to ensure consistent meshing and accurate results.

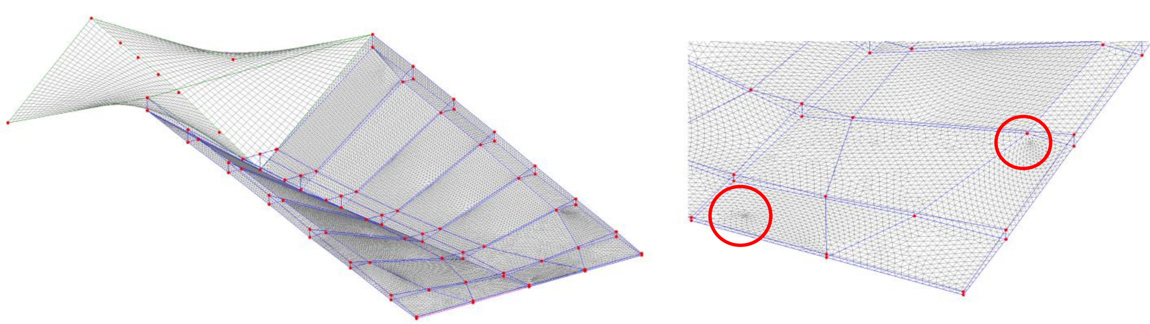


Figure 6.7: Irregular Geometry showing mesh concentrations and asymmetry.

Once the meshing was smoothed, thickness and material were assigned to each surface. These properties were also visualized by the model, which showed some of the thicker elements overlapping each other spatially, especially when the thickness was different between two connected elements. Although this approximation looks odd, it does not pose any problems for the solver, which is solely calculating stresses and displacements in each element as they are transferred from node to node. When the surfaces had been completed, supports were added to specific points, which in all cases correspond to the corner points of surfaces that form the roof. The final step in setting up the modeler was to add load cases, which were assigned to each specific surface. Details concerning each of these attributes are discussed in a later section.

The second part of the LUSAS software is the solver. Once the model was prepared, it was transferred to the solver, which completed the linear analysis. Although it is possible to output the raw results of the analysis, this was not useful since the more complex models contained over 50,000 equations. Instead, the solver is able to plot contours of a variety of different outputs such as stresses, strains, and displacements, and overlay them on the geometry of the roof. These contour plots, which also give maximum and minimum values, are combined with discrete value outputs of reactions at the supports to provide all of the content for the results section.

6.4. LUSAS Characteristics

LUSAS is a finite element analysis software designed to solve both linear and nonlinear stress, dynamics, composite, and other engineering analysis problems (*LUSAS*). Since it has many different applications, there are a few software options tailored to specific fields. This analysis uses the LUSAS civil and structural package, which includes the ability to analyze thin shell elements. Thus use of LUSAS was recommended by various members of the Princeton Civil and Environmental Engineering department due to its accurate solver, easy-to-use interface, and ability to mesh geometry directly in the modeler, removing compatibility issues that may arise from importing mesh from a different software. For all of these reasons, LUSAS was chosen to analyze the Miami Marine Stadium.

6.4.1.Element Choice

Since LUSAS is an upper level software, it is able to mesh the geometry of a structure automatically according to a few user-defined inputs. This allows a LUSAS user to experiment with many different types of mesh, even for the specific case of thin shell elements. For example, LUSAS is able to model triangular or quadrilateral thin shell elements, and it can interpolate between these elements either linearly or quadratically. It can also automatically generate transition patterns and irregular mesh for more complex shapes, although the irregular mesh must be closely monitored to ensure that concentrations of very small elements do not build up in one specific place. Given these possibilities, it is important to select the type of element that will yield the best results.

An extensive evaluation of the accuracy of different types of thin shell mesh including a convergence study has already been completed by Pazdon (GS ’09), and such an analysis is outside the scope of this thesis. However, a brief comparison between the two basic shapes (triangular and quadrilateral) available for modeling for the Miami Marine Stadium will be given here. It is generally accepted that quadratic mesh is more accurate in determining the actual behavior of a thin shell structure (Computers and Structures, Inc., 2004), and would consequently be the first choice for a finite element model. However, in the case of the Miami Marine Stadium roof there are certain places in which quadrilateral mesh requires highly irregular shapes, which could lead to large warping. For quadratic meshing to be accurate, it also requires that the four points defining each element must be co-planar (Prevost, 2012), which would be difficult to accomplish given the shape of the stadium. For a quicker yet reasonably accurate model, it may be more useful to use triangular mesh. To compare between the element types, one of the four models (FEM-2) used in this thesis was meshed in four different ways: quadrilateral mesh with linear interpolation, quadrilateral mesh with quadratic interpolation, triangular mesh with linear interpolation, and triangular mesh with quadratic interpolation (See Figure 6.8 - Figure 6.9).

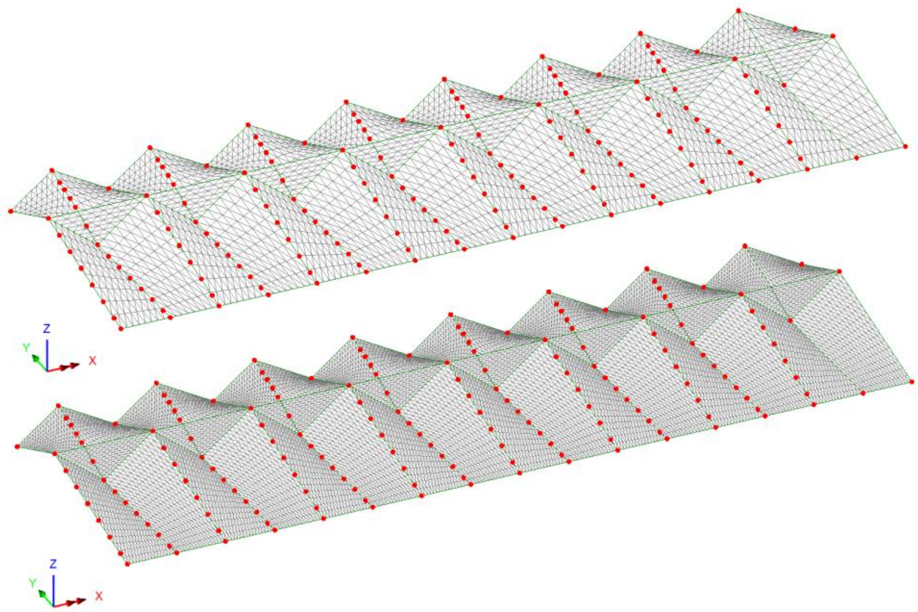


Figure 6.8: Stadium Roof with Triangular Meshing, 8 and 16 Local Divisions.

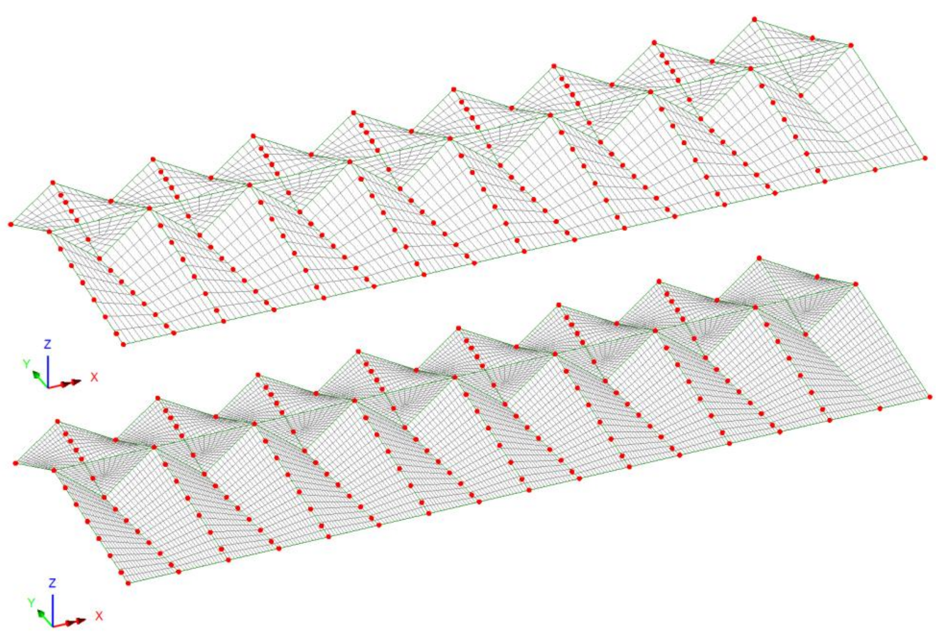


Figure 6.9: Stadium Roof with Quadrilateral Meshing, 8 and 16 Local Divisions.

These four types of mesh were run at two different degrees of fineness to analyze how certain outputs changed as the mesh was refined, forming a simple convergence study. For each different case, the maximum displacement and positive stresses in the cantilever direction are presented to test the accuracy of each mesh. The results of this brief study are given in Table 6.2:

Table 6.2: Comparison of different mesh patterns on Miami Marine Stadium Roof (U.S. Units)

		8 Local Divisions in Mesh		16 Local Divisions in Mesh	
Element Type	Interpolation Order	DZ - Max	SY Max	DZ - Max	SY Max
Quadrilateral	Linear	-0.06506	54.2298	-0.06556	66.2977
Triangular	Linear	-0.05291	52.3144	-0.06878	69.0602
Quadrilateral	Quadratic	-0.08002	87.5003	-0.07894	85.6752
Triangular	**Quadratic**	**-0.07881**	**82.2107**	**-0.07882**	**84.8358**

As can be seen in the table, the results were erratic when the interpolation order was linear. In these cases, the results given for displacements between the quadrilateral and triangular mesh are 19% different in the larger mesh and 4.9% different in the finer mesh. When the interpolation order is linear, this difference drops to 1.5% in the larger mesh and 0.2% when the number of local divisions per element is doubled. Furthermore, the maximum displacement measured in the triangular, quadratic mesh remains effectively constant with the refinement of the mesh, suggesting this is reaching a convergence point. Although the maximum stresses do not appear to converge, the maximum displacement is a more widely accepted metric for evaluating accuracy of the FEM model. In the case of this model, displacement is more accurate as a metric partially because as the mesh is refined, it is more likely for smaller whole elements to be located inside of a stress well. Since the model does not account for the steel and extra concrete at the locations of these stress wells, the maximum and minimum stresses become artificially high in magnitude. This tendency can be seen in the trend towards increasing stress across the different element types as mesh was refined, particularly when interpolated linearly. This trend continued when the number of local divisions was increased to 32 for a select few models.

Although this data clearly shows that the elements should be interpolated quadratically for the highest degree of accuracy, it is necessary to choose between quadrilateral and triangular mesh for the analysis of the stadium. Of these options, the quadrilateral element (QSL8) is a semi-loof, isoparametric element, meaning that one set of shape functions is used to define both the element's geometry and its displacements (Pazdon, 2008). However, as noted by Horstmeyer (B.S.E. '10) in his FEA analysis of glass hypar umbrella, the nodes of QSL8 behave like hinges in that no rotational fixity can be prescribed as a result of their semi-loof property (Horstmeyer, 2010). When modeling cantilevers, this could lead to inaccurate results. Furthermore, since automatic meshing was used to develop a warped shape, the need for planar nodes in a quadratic element reduces its usefulness. For these reasons, a triangular, quadratically interpolated meshing pattern was chosen. LUSAS names this element TSL6, which is a 3 node element with 6 degrees of freedom per node and considers both membrane and bending behavior.

Although only the results of triangular mesh will be plotted, it is important to note that in each mesh case the stress pattern and deflections were very similar, even if the magnitudes of their minimums and maximums varied slightly. The purpose of this chapter is not to delve deeply into the intricacies of finite element modeling, but rather to use the modeling as a tool to understand the structural behavior of the stadium. Furthermore, the formal analysis of the stadium is comparative, and consistent meshing is used across the FEM models to determine where their behavior differs. Based on the meshing and element recommendations of other studies, as well as the demonstrated convergence of the triangular meshing, the models displayed in this chapter will adequately accomplish the purposes of understanding form and providing a comparative analysis.

6.4.2.Linear Analysis

The structural study is completed using linear finite element analysis, which assumes that all materials are linear elastic and deformations are small (*LUSAS On-line Reference Manual*).

Although linear analysis ignores such possibilities as buckling, structural cracks, and permanent deformations, it is reasonable to simplify and assume linear behavior for the majority of engineering applications. This analysis assumes internal stresses in each element will vary linearly, membrane and flexural deformations are taken into account, transverse shearing is ignored, and each shell element is analyzed through Gaussian integration.

6.4.3. Material Properties

This stadium roof was modeled as a continuous thin shell made entirely of concrete, ignoring the steel reinforcing in the shell elements, but including its weight. Different thicknesses were assigned to specific portions of the roof as dictated by the construction drawings. LUSAS provides a library of common building materials that includes concrete, although some of concrete's default properties had to be modified in LUSAS to match the specific type of concrete dictated by the stadium's designers. The construction drawings specify that Class 4000 (psi) Lightweight Concrete be used for the main cantilever, but that Class 4000 (psi) Regular Weight Concrete be used on the backspan to balance the weight. Table 6.3 gives the material properties assigned to each of these regions of the model:

Table 6.3: Material Properties of Concrete Used in Stadium

Property	Lightweight Concrete	Normal Weight Concrete
Strength	4000 PSI = 27.57 MPa	4000 PSI = 27.57 MPa
Grade	Custom	AASHTO LRFD 4th 4.0 KSI
Young's Modulus	524,224 PSI = 3,614 MPa	524,224 PSI = 3,614 MPa
Poisson's Ratio	0.2	0.2
Density	1601 kg/m^3	2402 kg/m^3
Coefficient of Thermal Expansion	6.0 x 10^{-6} K^{-1}	6.0 x 10^{-6} K^{-1}

6.4.4. Design Simplifications

Since the Miami Marine Stadium roof is not a simple shell and has a variable cross section near the groins and supports, it is too complex to analyze as shell elements with perfect accuracy. As such, a number of simplifications were made in order to make analysis possible.

First, the roof is conceptualized as being made of entirely thin-shell elements connected at the nodes. Although an element may have a different thickness than the element beside it, each element has a constant thickness and is assumed to behave like a thin concrete shell, even in the thicker regions of the stiffening diaphragm and backspan. Perfect performance of concrete is also assumed, although in practice this is largely dictated by the skill of the contractor who places the concrete. In addition, smaller functional elements of the stadium, such as the roof hatch and the hanging press box, were removed for analysis. In the actual stadium, there is also a thick edge brace that runs along the sides of the stadium cantilever, supporting the edges. Although the presence of an edge beam affects the structural performance of the stadium, only the shell elements were modeled, and this must be taken into account when viewing stress plots. For example, maximum tensile stresses in the model occur where an edge beam would be located in the existing structure, so to some extent only the interior behavior of the shell is most important.

6.4.5. Boundary Conditions

Although in reality there is some give to the roof supports since they sit on top of long columns, the supports are assumed to be fixed. This more accurately represents the conditions of the roof than assuming either of them is pinned, because there is substantial interplay between the reinforcing of the roof and the columns where they are connected inside a mass of concrete at the supports. Where thin shell elements of different regions (such as the stiffening diaphragm to the cantilevering section) intersect, special care was given to ensure connections between the nodes of elements all along the boundary so that the entire weight of the roof is not being transferred to the supports through a single element. Similarly, all of the eight umbrellas are connected along their boundary edges, mimicking the presence of steel weld tabs and making it so that each umbrella does not stand alone. All of the cantilever edges that are not connected to another element are left free.

6.4.6.Loading

None of the original calculations done by the stadium's engineer still exist, so the exact nature of loads that were calculated is unknown. Therefore, for a comparative analysis of structural performance, this thesis assumes only the dead loads contributed by the self-weight of the concrete and steel in the stadium roof. The weight of the concrete is distributed and assigned according to the actual thickness of concrete at any given element of the model, and is thus not constant. The weight of the steel, though almost negligible when compared to the concrete, was added as a constant distributed load acting over the entire area of the roof. The weight of the steel was determined by estimating the amount of steel reinforcement present in the construction drawings. Since there is considerably more steel in the backspan than in the main cantilever, the roof was broken into two sections at the diaphragm and higher steel loads were added to the backspan only.

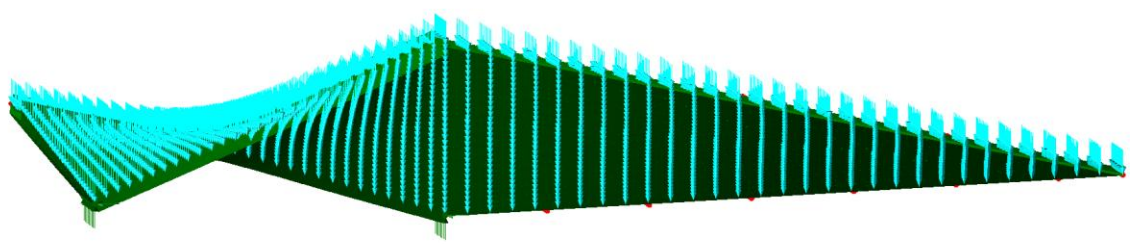


Figure 6.10: Visualization of gravity loading on the shell roof. The greater density of arrows on the backspan show there is more weight located in that region of the structure.

LUSAS realizes the self weight loads of the concrete by assigning a prescribed downward acceleration in the vertical direction equal to acceleration due to gravity (See Figure 6.10). Since the density of concrete is stored as a property of each element along with an element's size and thickness, LUSAS is able to automatically determine the gravity loads of the entire structure as a result of this prescribed acceleration. This option is called a 'body-force' load in LUSAS, and the manual instructs users to apply gravity dead loads in this manner for the most accurate results. The additional steel loads are globally distributed, force per unit area loads acting in the negative vertical direction over the entire roof. These steel loads amount to 0.201 kPa over the backspan and 0.110 kPa over the main cantilever. Normal reinforced concrete assumes a value of 25 kN/m^3, which gives a value of 1.9 kPa for the stadium roof when it is adjusted for the thickness of the shell. Although the steel loads are substantially lower than what is normally used, the estimates are based on calculations of how much steel is present in the actual construction drawings, so they are likely not as conservative as what is traditionally used. Furthermore, the magnitudes of these loads are so small in comparison to the weight of the concrete that they will not significantly affect the overall results.

6.5. Results

6.5.1. FEM-1

6.5.1.1. Validation

Adriaenssens *et. al.* (2012) replicates the simplified mathematical analysis completed by Meyer for the load case of self weight in the Miami Marine Stadium, which is applied in the finite element analysis as well. Since both methods use the same load case, the hand calculations can be used to validate the FEM models. Adriaenssens *et. al.*'s calculations make a number of assumptions: One-way slab action between valley and ridge folds in the transverse direction, and longitudinal beam action of the plate parallel to the folds in the longitudinal direction. The one-way continuous slab approach in the *x*-direction reduces moments at the midspan, while also causing negative bending moments (See Figure 6.11). In the *y*-direction, the roof acts as a cantilevered beam with two supports, showing the largest moment on the interior support (See Figure 6.11). To accurately calculate the cantilevered beam action, the beam was divided into 10 segments, with the profile properties changing for each segment due to the varying cross section of the groins. The cantilevered beam is loaded with the support reactions calculated in the one-way slab analysis, which in turn leads to stress results from the bending moments present in the beam (Adriaenssens *et. al.*, 2012). In addition to the stress results, this thesis used a similar method to determine the reaction on the interior column for comparison with the value given by FEM-1. The calculations for the simple method are provided in Appendix C.

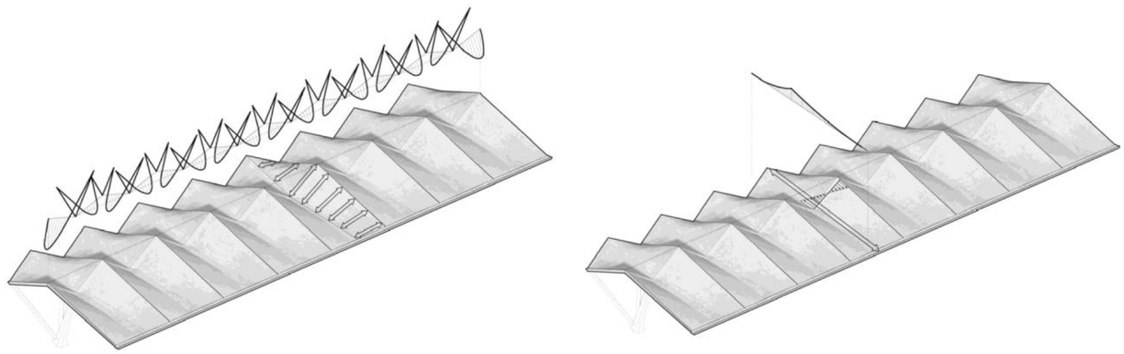


Figure 6.11: In the *x*-direction of the span, the thin shell sections between groins act as a continuous plate supported by the folds (Left). In the y-direction of the span, the roof acts like a cantilever sitting on two supports. Images courtesy of Sigrid Adriaenssens.

According to the hand calculations, the axial force in the groins is obtained by dividing the calculated beam moment by the depth between the ridge and valley groins. At the interior column of the stadium, the maximum bending moment of 4,280 kN*m results in an axial compressive force of 823 kN. Taking into account the area of the fold groin, the groin stress comes to 2.1 MPa (Adriaenssens *et. al.*, 2012). As can be seen from the stress plots for FEM-1 (Figure 6.12), this is very close to the stress magnitude near the shell support calculated through finite element analysis, neglecting the stress well. Furthermore, the total reaction on a central support of the stadium calculated by a simple folded plate method is 4168.0 kN, which is only 9.5% percent different than the values given by the FEM-1 model under the same load condition. Thus, the model is behaving in a manner predicted by hand calculations based on accepted historical methods for designing folded plate roofs. For the purpose of giving a comparative analysis, the folded plate calculations validate the model to a reasonable accuracy.

6.5.1.2. Stresses

In order to study the behavior of the roof, the stresses in the direction of the cantilever (SY in each of the models) are given for the top and bottom of the thin shell elements in Figure 6.12. Maximum and minimum stress values are shown on the plots and listed in Table 6.4.

The model shows behavior typical of hypar umbrellas under self-weight, especially in the much thinner cantilevered region. As predicted by the simplification of an umbrella as a cantilever triangular in section, compressive stresses of up to 2 MPa develop in the valleys of the roof, while the peaks of the roof are in tension at a similar magnitude. The magnitudes of these maximum and tensile stresses are on the same order, although the compressive stresses are slightly higher throughout the stiffened concave groins. The higher stresses in thicker regions seem counterintuitive, since there is more material at the groins to take the loads. However, as shown by the cantilever analogy for a hypar umbrella, the forces are primarily flowing through these groins to the support. Since the peaks are the highest point of the roof and are attached to both the stiffening diaphragm and the backspan roof, they are being pulled on by the weight of the roof and are resisting the moment of the cantilever through tension. The opposite is true for the valleys, which are taking the weight of the cantilever in compression down through the main support column. Although this model only considers gravity loading with no safety factor, neither compressive nor tensile stresses reach higher than the reduced ultimate strength of 13.1 MPa in these regions. This reduced strength is based on the type of concrete specified for the construction drawings. Low stresses of less than 1 MPa are encountered in the backspan, which is much smaller, thicker, and is supported on each corner of a given hypar form. The behavior of the backspan shows that the performance of the cantilever drives the efficiency of the design, since the weight of the shorter backspan is used to balance loads rather than reduce material.

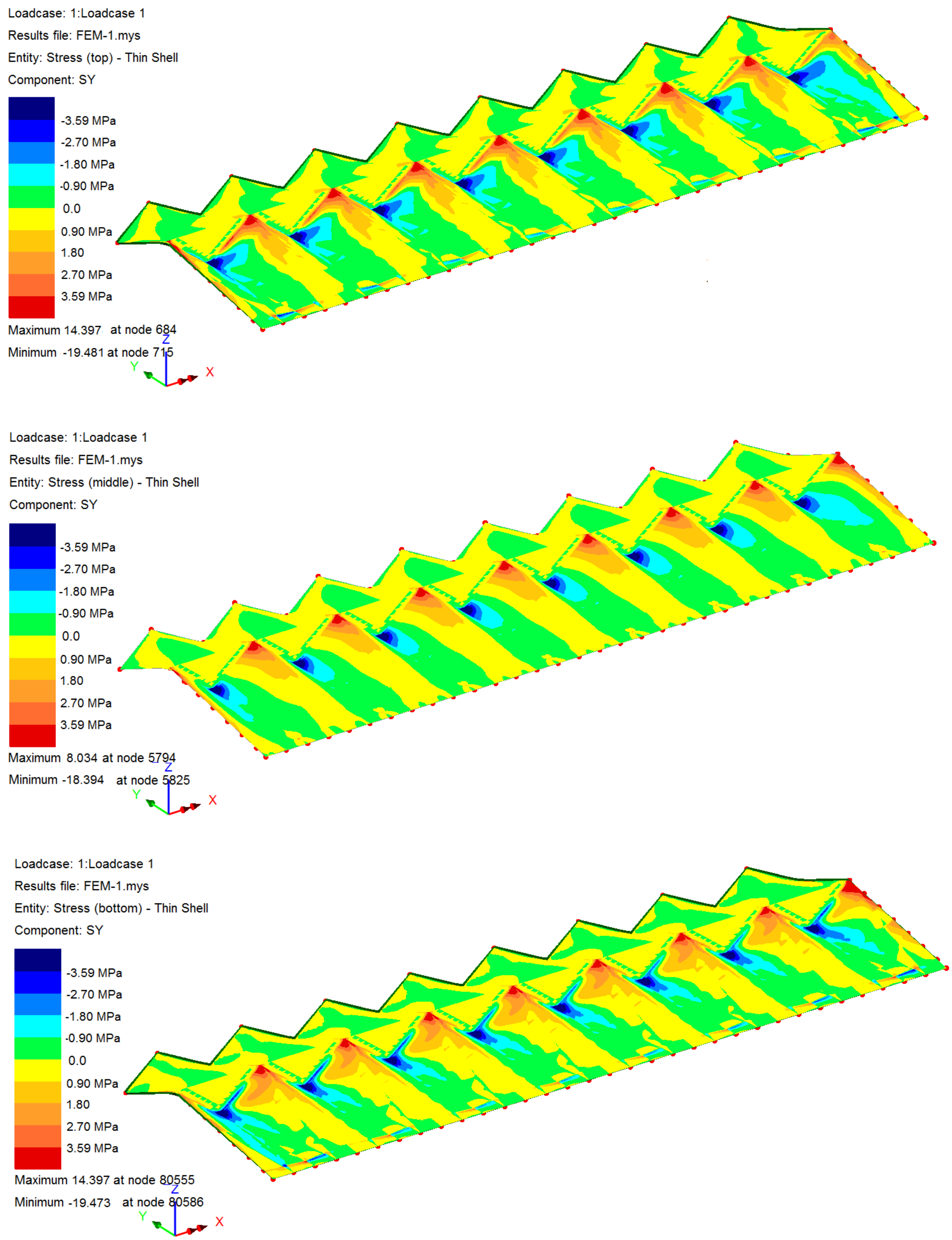


Figure 6.12: Element Stresses in FEM-1.

The FEM-1 plots give high values of up to 14 MPa for maximum tensile stress and up to 19 MPa for compressive stress, which is higher than the reduced ultimate strength of concrete. However, it can be seen that these high magnitude stresses occur only in a few stress wells located at the roof supports and peaks. The stress wells may be partially due to the geometry of the model at these points, which includes smaller surfaces and consequently smaller elements, since the number of local divisions in each surface's automatically generated meshing remains constant. Whatever the case, these stress wells can be ignored because they do not accurately represent the physical roof at the supports. In the actual stadium, the places where columns meet the roof contain much more concrete as well as a considerable amount of reinforcing steel, which works to directly connect the reinforcing patterns of both elements. Since this was not feasible to model but exists in reality, the stress wells shown on the plots should be neglected.

6.5.1.3. Deflections

The deflection pattern shows the roof to have gradually increasing negative displacements in elements as they get further from the supports. The front, free edge of the stadium has the largest deflections as expected, and the maximum deflections for the complete structure occur on the free side of the exterior umbrellas. If a slice is taken at a given coordinate in the *y*-direction (the slice would be parallel to the *x*-axis, which runs along the front edge of the stadium roof) the deflections have an absolute maximum in the ridges of the cantilevered section, a local maximum in the valleys, and minimums at the midpoints between these two, creating a wave pattern when looking at the plot from above (See Figure 6.13). There is a slight positive deflection in the roof areas near the main umbrella supports, but this is negligible compared to the negative deflections in the cantilever. There are no noticeable deflections in the backspan of the roof.

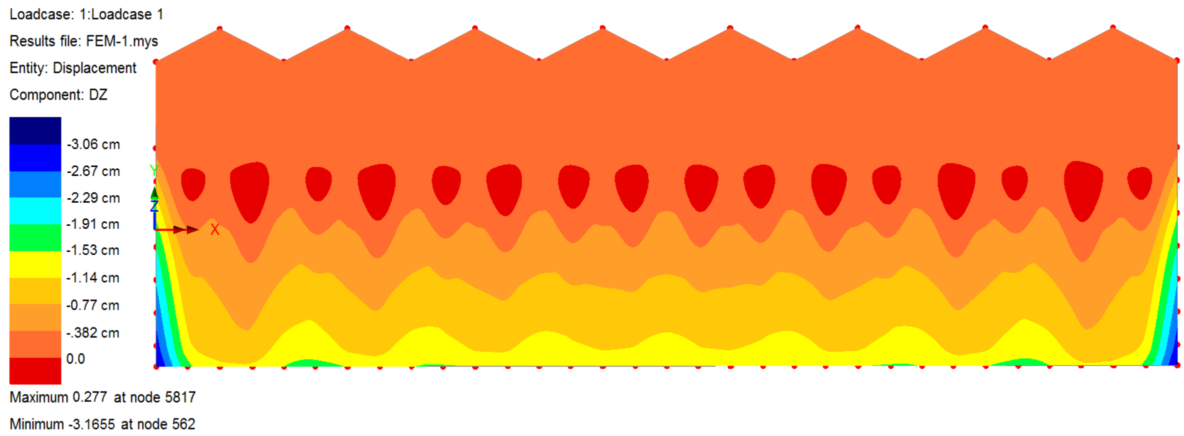


Figure 6.13: Displacements in FEM-1

The maximum deflection over the entire roof is 32 mm, which is considerably lower in magnitude than the allowable deflection of 1/150 of the span for this type of structure, which would be 13.47 cm. It is also a considerably smaller value than the actual deflections (15 cm) as measured by engineer Jack Meyer after removing the formwork. However, this model assumes perfect behavior of concrete without creep, which is impossible to achieve in reality. It also assumes linear elasticity and neglects permanent deformations, which are simplifying assumptions that could have affected the deflections analysis.

In order to more accurately compare the deflections to analytical calculations, these deflections were calculated using an additional live load created by weight placed on the stadium. These extra loads were calculated and applied by Simpson, Gumpertz, & Heger Inc. (1993) during an assessment of the structural health of the roof after Hurricane Andrew (Simpson *et. al.*, 1993). The Simpson *et. al.* report has slightly different values for the strength and weight of the concrete taken from samples of the stadium's actual material, which were also considered for this supplemental deflection analysis; a comparison between FEM-1's values and those used by Simpson are given in Appendix B. Even with this additional live load, the deflections for FEM-1 remain much lower than what was found through analytical calculations. The deflections for a middle umbrella of the main cantilever are given in Figure 6.14, and are shown to be less than 3 cm at the tip.

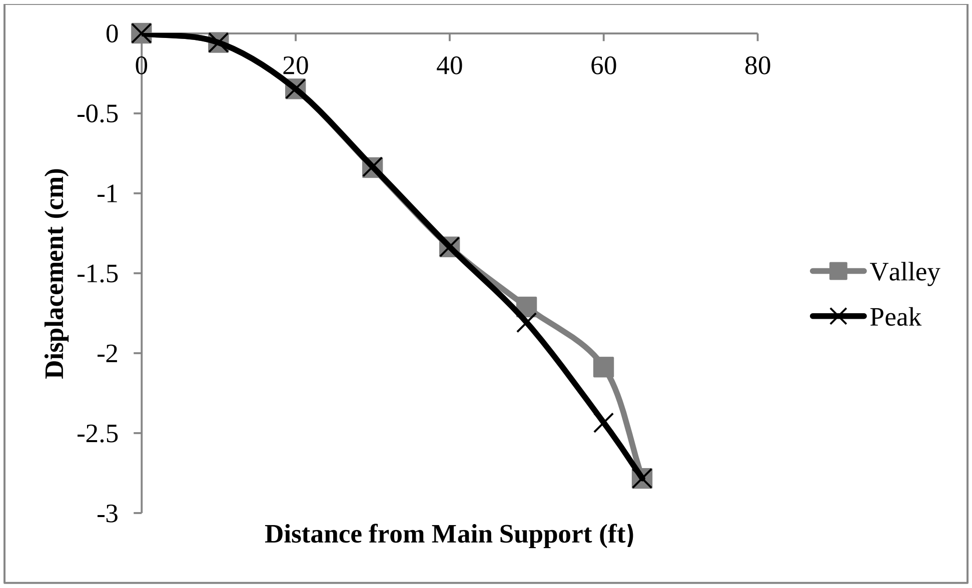


Figure 6.14: Deflections in a middle umbrella of the main cantilever, at discrete points along the roof.

6.5.1.4. Membrane Structure Performance

According to membrane theory, a thin shell should have stresses distributed equally along its cross section. Thus, the stress behavior should be the same for the top, middle, and bottom of each shell. The plots for all three of these cases in FEM-1 are similar, with stress wells at the peaks and supports and small magnitude stresses in both the backspan and the main cantilever (See Figure 16.12). Curiously, the top and bottom stress plots are almost identical even up to the magnitude of their maximum tensile and compressive stresses, although these occur at opposite sides of each model, on the leftmost umbrella for the top and the rightmost umbrella for the bottom. The middle of the shell is subjected to a much smaller maximum tensile stress (8.0 MPa to 14.4 MPa), but these occur in the stress wells and cannot be trusted. Overall, the roof exhibits reasonably good membrane structure performance, despite having thickened groins which shift the behavior of the roof more towards that of folded plate beams rather than thin shells.

6.5.2.FEM-2

Stresses

FEM-2 represents the stadium roof with no thickening in the groin vaults, meaning that the cantilever shell has a continuous thickness of 7.62 cm. As with FEM-1, the stresses in the direction of the roof (SY in each of the models) are given for the top and bottom of the thin shell elements in Figure 6.15. Maximum and minimum stress values are again listed in Table 6.4.

The stress pattern of FEM-2 shows that when the stiffening groins are removed, the higher stress areas are distributed further along the spine of both the ridges and valleys, but these stresses are on the same order (-2.87 MPa < x < 2.87 MPa) if slightly larger. As expected, the ridges of the cantilever section are in tension and the valleys are in compression. These stress values get higher closer to the supports, because they are made to resist stronger bending forces, analogous to the behavior of a triangularly shaped cantilever. Unlike in FEM-1, there are no stress wells in any portion of the stadium roof, since the maximum and minimum values (5.503 MPa and -4.116 MPa) are not substantially higher than stresses found throughout the thin shell region. The stresses in the backspan of the cantilevers do not reach above the first stress level of 0.90 MPa, as denoted by their continuous green and yellow color.

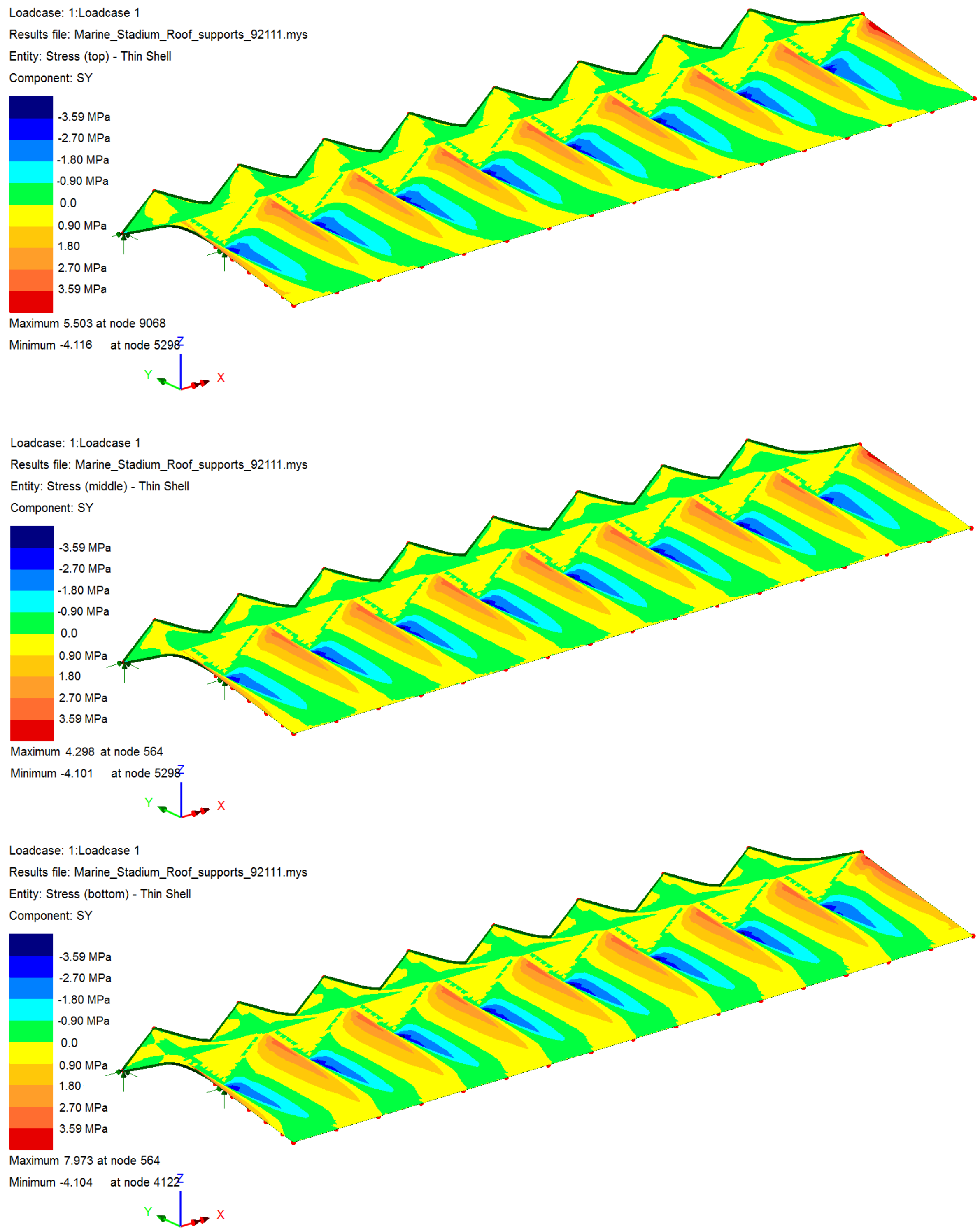


Figure 6.15: Element Stresses in FEM-1.

6.5.2.2. Displacements

The deflection pattern of FEM-2 under both dead and live load is almost identical to the pattern of FEM-1 in that the deflections in the negative z-direction gradually become greater towards the free end of the cantilever. However, FEM-2 reduces the maximum deflection by 17%, and this reduction is carried through the rest of the structure (Figure 6.16).

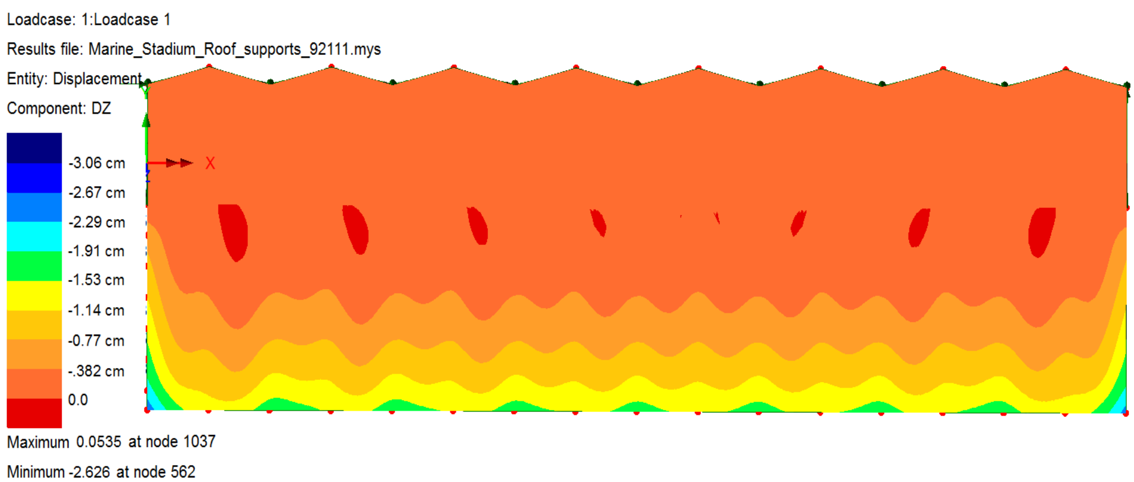


Figure 6.16: Deflections in FEM-2.

6.5.2.3. Membrane Behavior

Throughout most of the FEM-2 model, there is no significant difference between the top, middle, and bottom stresses, which suggests a high degree of membrane behavior. However, in both the backspan and the small region of the cantilever closest to the free edge, tension and compression are flipped between the top and the bottom. A possible explanation for this is forces in the *x*-direction between umbrellas. Two side-by-side umbrellas may be pushing on each other, causing a small ripple centered along the *y*-direction of this region. If this ripple exists, an artificial peak would be formed, causing the concrete to be in tension (and possibly crack) at the top of the shell and be in compression in the shell's bottom. This theory is supported by plots of stress in the *x*-direction, which show both a mirror pattern between the top and bottom of the shell

(See Figure 6.17) as well as stronger forces near the edge. Nevertheless, the rest of the shell exhibits good membrane behavior, which is to be expected from its hyperbolic umbrella shape.

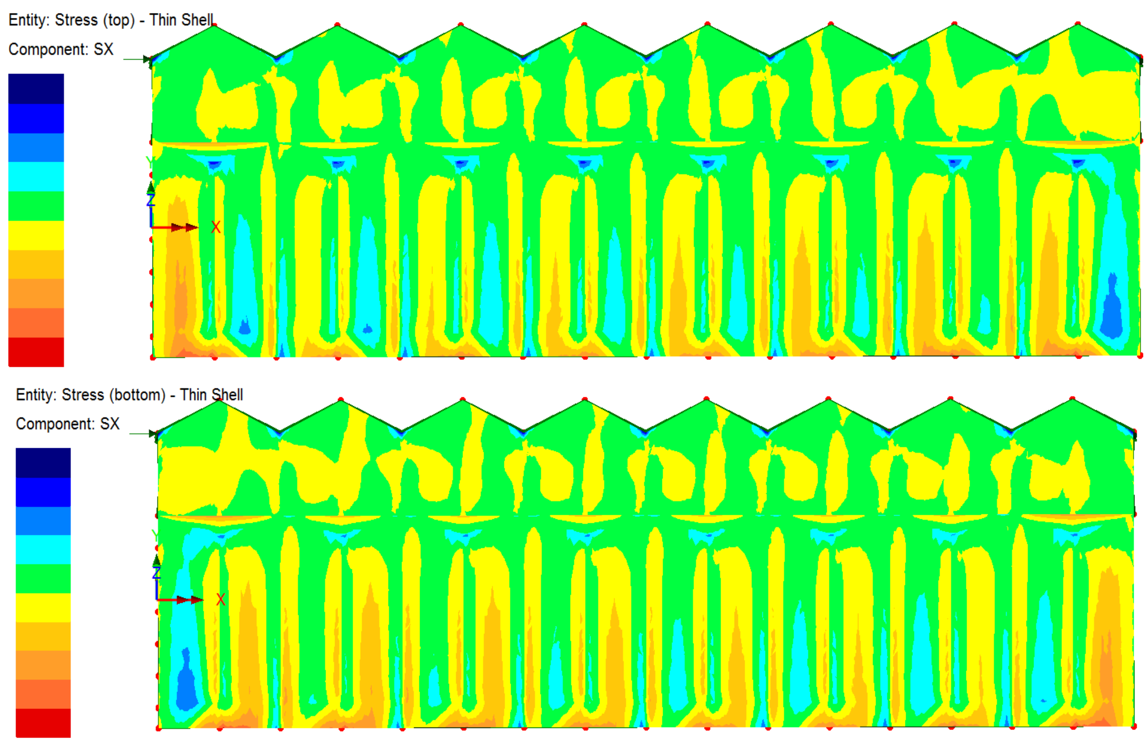


Figure 6.17: Element Stresses in the *x*-direction.

6.5.3.Models Without Stiffening Diaphragm

6.5.3.1. Stress Comparison in Variable Area Shells (FEM-3)

Model FEM-3 is identical to FEM-1, except that the stiffening diaphragm has been removed. Upon inspection of the stress plots for FEM-3, it is clear that removing the diaphragm does not substantially diminish the structural performance of the roof. For example, when the middle stresses in the shell are compared to the same plot for FEM-1, the distribution of stresses is nearly identical, as is the magnitude of these stresses. The maximum compressive stress does go up by about 10%, but the maximum tensile stress is actually reduced. In a pure folded plate structure, such a stiffening diaphragm would be expected to have a much more positive effect on the design. Since this is not the case, finite element modeling suggests the diaphragm is not needed, even in the original design including stiffening groins.

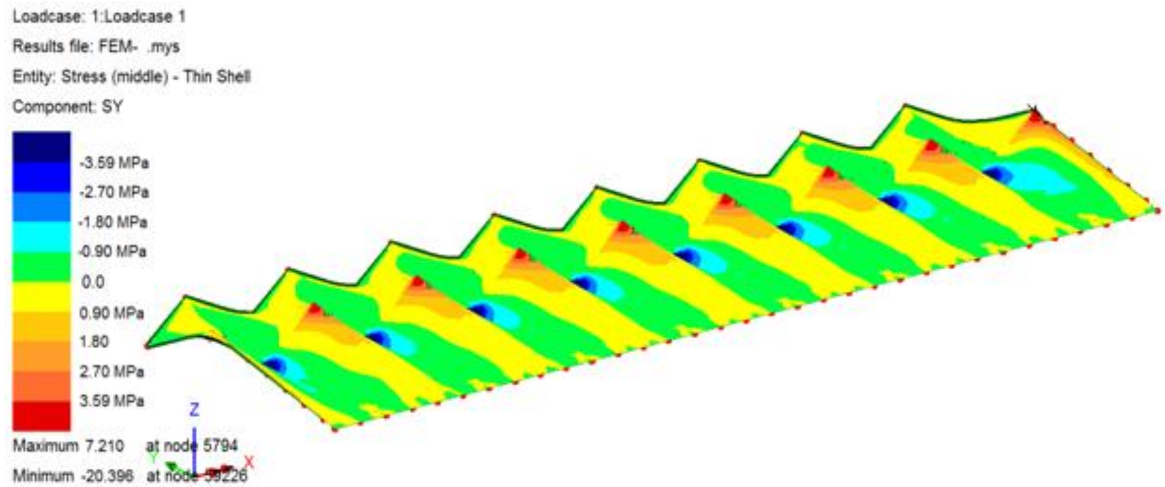


Figure 6.18: Element Stresses in FEM-3

6.5.3.2. Stress Comparison in Constant Area Shells (FEM-4)

The FEM-4 model has a continuous thickness in the main cantilever, but does not contain a stiffening diaphragm. The same is true for FEM-4 as was shown in the last section for FEM-3: removing the stiffening diaphragm has only a minimal effect on the structural behavior of the rest of the roof (see Figure 6.19). This time, the maximum tensile stress of the roof, which occurs on

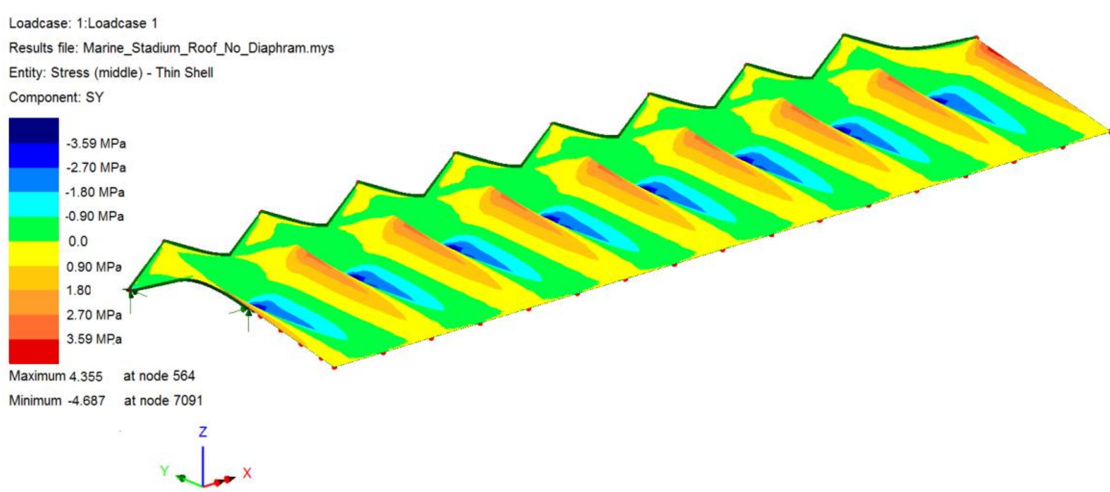


Figure 6.19: Element Stresses in FEM-4

the peak ridge lines of the cantilevered portion, is increased slightly. The maximum compressive stress due to self weight is increased by 14% to -4.101 MPa, but this is still well below the 13.1 MPa ultimate reduced strength of the concrete. The distribution of stress over the entire roof does not change enough from FEM-2 to FEM-4 to be detected by a visual comparison of each stress plot. Ultimately, analysis shows that in terms of stress, the diaphragm does not help the structural performance of the stadium. To confirm the lack of a performance gap between the stiffened models and the unstiffened models, however, it is also necessary to look at deflections.

6.5.3.3. Deflections in Models without Stiffening Diaphragm

In addition to stresses, the deflections of FEM-3 and FEM-4 were also analyzed to understand how the shape of the shell roof behaves structurally without the existence of the stiffener. In folded plate structures, the diaphragm is meant to hold the folds in place, which would theoretically cause a large decrease in deflections in the main cantilever for the stiffened models. Thus, large differences in deflections between the first two FEM models and their counterparts with the diaphragms removed would indicate the usefulness of the folded plate elements. To test whether or not the models confirmed this behavior, the deflections for each FEM model under dead load was calculated along a central groin of the cantilevered umbrella. These deflections do not consider the additional live loads applied by Simpson *et. al.*, and are thus lower than the deflections presented in previous sections. However, a comparison between them is valid because the loading conditions were kept constant. Figure 6.20 shows the deflection results for FEM-1 through FEM-4. In general, a more tightly grouped deflection pattern would indicate the intrinsic ability of the hypar form to prevent against deflections.

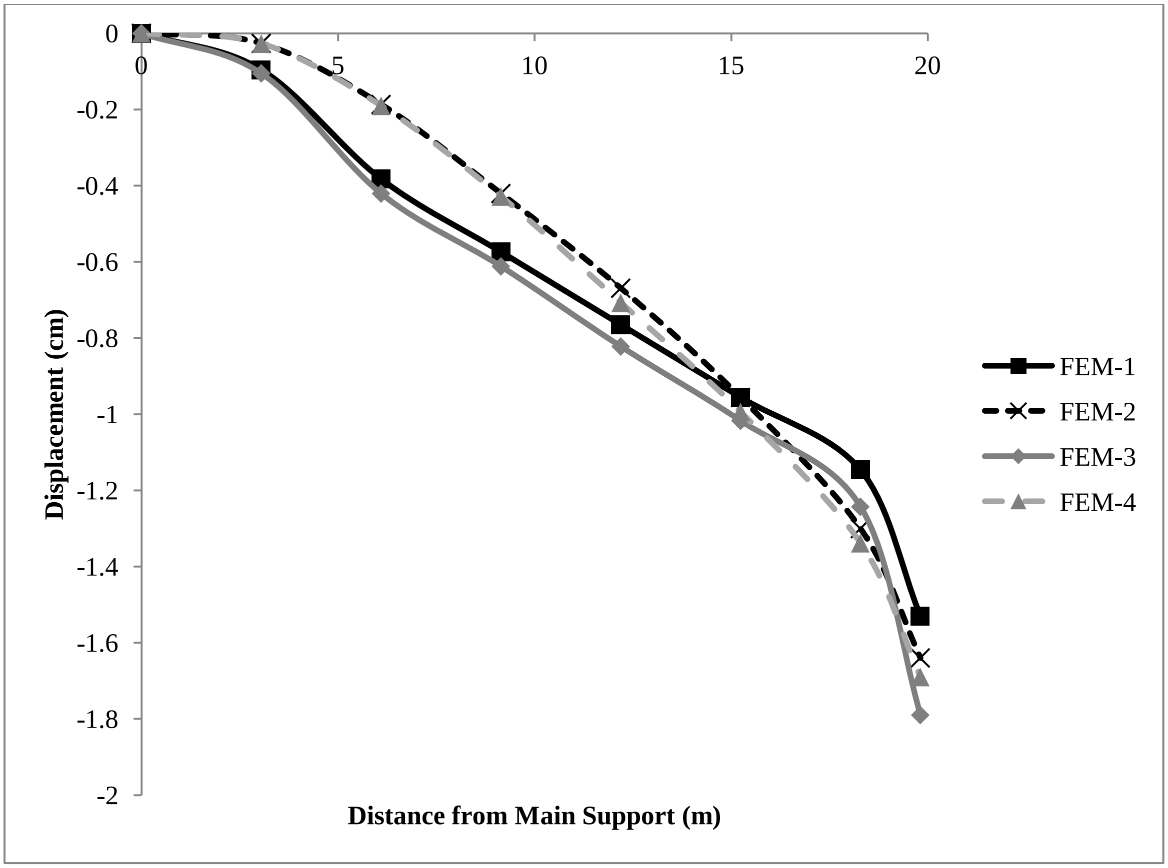


Figure 6.20: Deflections in the center of an umbrella gradually increase towards the front edge, but do not change substantially with the removal of the stiffening diaphragm.

These plots show that the deflection pattern remains similar for each of the four models, whether or not the stiffening elements are included. Although FEM-1 exhibits the smallest tip deflection and FEM-4 gives the highest result for deflections at this location, the difference in deflections is never more than 2 cm, even at the front edge of the main cantilever. The existence of the diaphragm does have a more positive influence on deflections than does the stiffening in the groins, since the curves for FEM-3 and FEM-4 are both lower than for FEM-1 and FEM-2. However, compared to the scale of the stadium, none of these differences are significant. In terms of deflections, the roof could have been built without stiffening, which would have reduced reactions and material use while still maintaining safe and serviceable deflections.

6.5.4.Results Summary

All four FEM models show behavior corresponding to the expected performance of thin shell hypar umbrellas. Umbrella behavior is seen only in the main cantilever portion of the roof, since the backspan is considerably thicker and supported on multiple corners, causing its form to behave more comparably to an arch than a cantilever. In each of the models, the valleys of the roof are in compression, while the peaks are in tension. Since their behavior is so similar, the different models can be compared directly with a number of different performance indicators, which are given in Table 6.4 and Figure 6.21. The first performance indicator, material reduction, is indirectly related to both cost and embodied energy in construction, since less material reduces both aspects of the structure. In this comparison, FEM-1 is taken as the baseline because it most closely resembles the stadium as it was built, and the other models are situated as hypothetical improvements to the design. The other two indicators, stress and deflections, compare the structural efficiency and safety of the stadium roof.

Table 6.4: Structural Properties of the Various Finite Element Models under Dead Load

Model	Material Reduction	Range of Typical Roof Stresses in Backspan	Range of Typical Roof Stresses in Thin Cantilevered Portion	Deflection at Middle Cant. Tip
		(MPa)	(MPa)	(cm)
FEM-1	0%	-0.90 < x < 0.90	-1.80 < x < 1.80	1.53
FEM-2	-27%	-0.90 < x < 0.90	-2.87 < x < 2.87	1.64
FEM-3	-5%	-0.90 < x < 0.90	-1.80 < x < 1.80	1.79
FEM-4	-32%	-0.90 < x < 0.90	-2.87 < x < 2.87	1.69
Note: All Stresses measured in the middle of the shell in the direction of the cantilever (SY). There were differences in stress between the top/bottom of shell and the middle, but for the most part these differences were minimal as the material exhibited membrane behavior.				

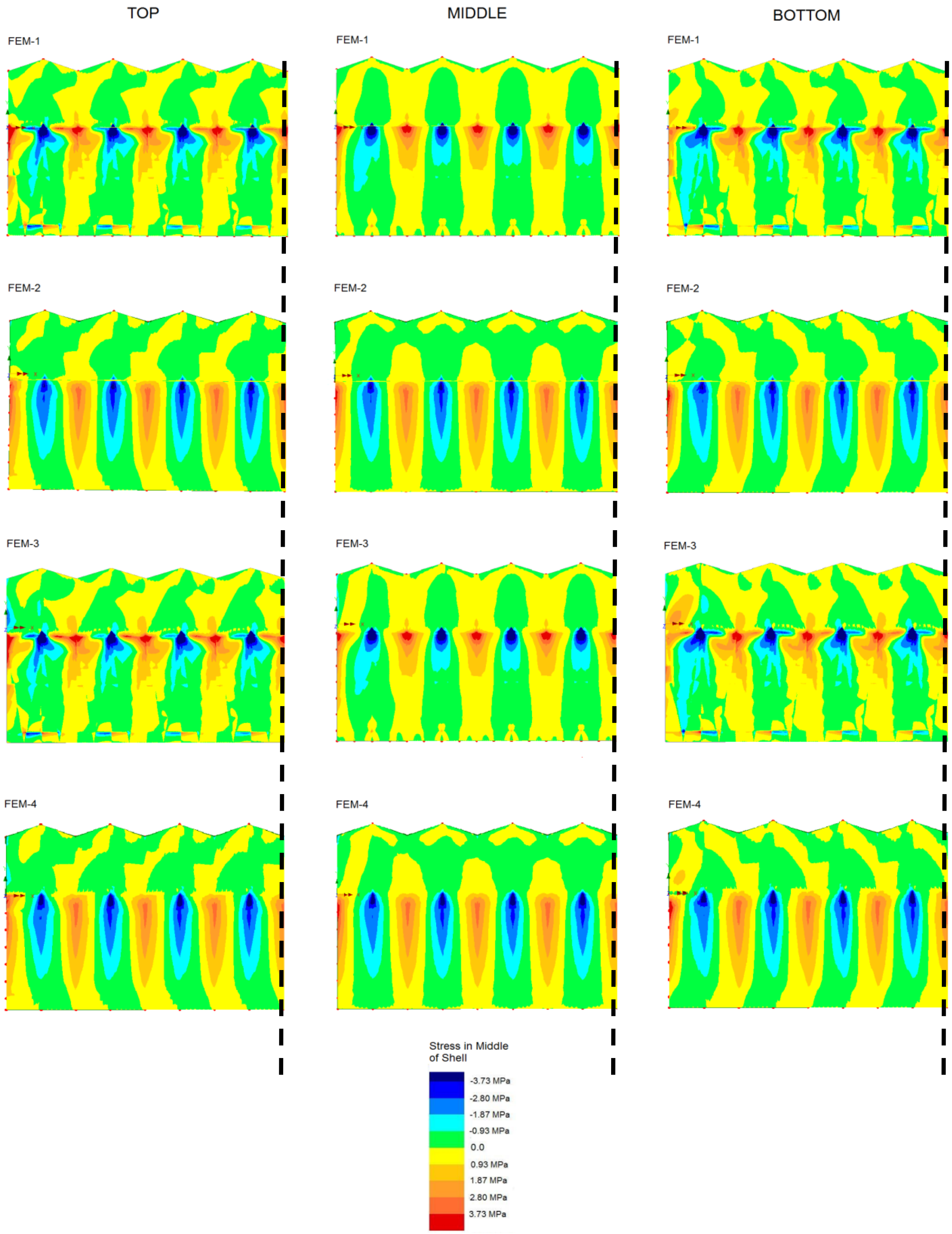


Figure 6.21: A comparison of the stress patterns for each FEM model, showing shell behavior in the top, middle, and bottom of the shell.

Broadly speaking, the results of the Finite Element Analysis show that Meyer produced a well-performing structure in his design of the Miami Marine Stadium roof. Despite its complex shape and the lack of advanced analysis tools available to him at the time, Meyer was able to realize Candela's proposed geometric form by building a structure with minimal deflections and stresses well below what is acceptable given the strength of reinforced concrete. Furthermore, he was able to largely balance the asymmetrical cantilever by using normal weight concrete in the shorter backspan and lightweight concrete in the longer main cantilever, which proved to be a stroke of engineering ingenuity. The fact that the stadium has stood through hurricanes is a testament to this ingenuity, as well as to Meyer's ability as an engineer. To verify his design, Meyer made a series of conservative analytical calculations, while also choosing to add stiffened groins and a diaphragm, both of which were commonly used in similar structures in the 1960s. An attempt to replicate Meyer's calculations (Adriaennsens *et. al.*, 2012) has been shown to match the results obtained by finite element modeling in terms of reactions and stresses with reasonable accuracy, illustrating the value of Meyer's hand calculations. In his time and place, much more was known about the behavior of folded plate structures than hypar umbrellas, and thus Meyer's design was a safe and viable solution.

However, the FEM roof models presented in this chapter make it clear that the stadium could have been designed as a series of constant thickness, monolithically connected hypar umbrellas and constructed with a considerable reduction of material. When the stadium roof is modeled as a thin shell with a continuous 7.62 mm thickness, the deflections do not change much when compared to the original design, owning to the drastically smaller amount of self-weight the roof must support. The stresses in the ridges and valleys of the FEM-2 design are higher than in FEM-1, but are still well within a reasonably safe design limit. In addition, the natural shape of the roof allows the diaphragm to be removed without substantial negative effects, leading to even more material reduction and possibly a more elegant form. Given what the models have shown about stresses and deflections with the removal of stiffening elements, the designers could have retained the beautiful appearance of the stadium while saving almost a third of the material needed for its construction by adhering more directly to the hypar shape. With a purely hypar umbrella design, the stadium would have more clearly expressed its flow of forces, while also cutting an altered, curved profile against the skyline when viewed from far away. This finite element modeling exercise for the Miami Marine Stadium thus confirms the usefulness of the hypar form in terms of both its visual and structural properties.

7. Design Application of the Hyperbolic Paraboloid

7.1. Introduction

The previous chapters have shown thin shell concrete structures built in the hypar form to be geometrically versatile, environmentally sustainable, and structurally efficient solutions to a variety of design problems. This chapter applies knowledge of the hypar form and its built manifestation in a contemporary, developing world context by proposing a conceptual design for a hypar shell structure. The planning behind this structure attempts to be holistic, beginning with the framework of a real world social issue and keeping in mind social, economic, environmental, and physical constraints throughout the design process while determining how a structure can help to address the initial issue. Despite attempts to be holistic, the proposed design is in no way exhaustive from an engineering standpoint. A complicated structure such as the one proposed here would require a considerable amount of work on the details before it could actually be built. With many of these details, a conceptual discussion of how they could be addressed in the future will be included, since it is useful to catalogue design ideas even if they have not yet been fully developed. Despite these limitations, this design proposal can be considered a practical, viable thin shell design in its early stages, opening the possibility of future work on the design.

7.2. Detailed Problem Description

Due to broad lifestyle changes of the world population associated with globalization, a lack of physical activity has become a health issue across many different nationalities, ages, and economic statuses. According to a study by the World Health Organization in 2004, “physical inactivity is recognized as an important risk factor for multiple causes of death and chronic morbidity and disability” (Bull *et. al*., 2004). Although there is considerable variability in levels of physical activity, and as one might expect Americans and Europeans have the highest rates of being inactive according to standard definitions, this problem is not restricted to wealthy nations or crowded cities. In 2008, a research team led by Regina Guthold confirmed the widespread prevalence of physical inactivity in developing countries quantitatively by completing a survey of physical behavior in 51 nations across the globe. In Guthold's study, it was found that nearly a fifth of all people surveyed in the developing world suffer from inactivity. Women are disproportionally susceptible to inactivity, as are people living in cities, since inactivity was 3% higher for urban women and 6% higher for urban men than their rural counterparts. This study focused mostly on adults and bracketed results by age group, showing that inactivity increased with age (Guthold *et. al.*, 2008). However, despite these differences across demographics, it is important to stress that inactivity can be a health problem for entire communities, and the encouragement of healthy exercise habits in both children and adults can help to relieve health problems for people later on in life (Bull *et. al.*, 2004).

In general, inactivity is a function of complex social and economic factors. However, for some places, a lack of physical space suitable for exercise may contribute to the problem. The possibility of a lack of adequate exercise space in a particular region was noticed by the author during a trip to Shell, Ecuador in 2010. Although changes in altitude and proximity to the coast cause Ecuador to have a diverse range of climates, certain areas of the country have climate characteristics that make it difficult to consistently exercise outside. First, Ecuador sits right on the equator, meaning that there is considerably more exposure to direct sunlight and its associated risks throughout the year. When it is not sunny, it is usually raining; average rainfall in the Amazon basin region of Ecuador is as high as 368 cm per year, although it is less than a third of that in the Andean region (*climatemp.info*). During work with the *Casa de Fey* orphanage in Shell, which is on the boundary between these two regions, the author noticed that almost all physical education classes held by the orphanage took place under an open-air roof. This structure was centrally located at a common space in town, referred to as the *bodega*. The roof protected the children from direct sun and rain while they were exercising, and was even used on cloudy, dry days because the ground was wet from previous rain everywhere else except for under the roof. According to Guthold's study, physical inactivity rates for Ecuador are average for men (17.9%) and above average for women (27.7%), showing that many people in Ecuador will be at risk for health problems as they grow older if healthy exercise habits are not established during their youth and maintained throughout their lives (Guthold *et. al.*, 2008).

Given this personal experience, it is clear that many communities in Ecuador or similar climactic regions in Latin America could benefit from the construction of an open-air roof to house usable space for exercise. The community would have to be large enough to make building the structure financially worthwhile, but it could be completed in a town or a more urban context while addressing the same social issue. If desired, the structure could be built adjacent to a school or community center, or be shared by a number of comparable entities present in a community. Ideally, this structure would have a large enough free span to cover a small walking or jogging track for use by all ages and an interior multipurpose space that could be used for basketball, indoor soccer, and other forms of exercise. The roof would have to protect its patrons from direct sunlight and rain, although it could use these natural elements to its architectural advantage. In order to be successful, the structure would have to be environmentally, economically, and socially sustainable. In other words, it must be cheap, easy to build, and be constructed efficiently out of common materials that can be obtained locally. The construction procedure must be low-tech in order to reduce cost and allow the local community to contribute to its creation by providing labor. It is also important that the structure fits in with the cultural aesthetic sense of the people living there, since the roof would be of no use if people did not like or want to use it. Of course, even a structure fitting all these design constraints will not singlehandedly create a culture of healthy exercise and reduce citizen inactivity by itself.

However, if constructed as part of a holistic, community development focused initiative, an open-air exercise structure could contribute to making a community more livable, while also encouraging a healthy lifestyle among residents and their children.

7.3. Project Presentation

Given its inherent geometric and structural properties, a hypar shell structure could be used for the design application of an open-air exercise roof. Renderings of the conceptual design proposed by this thesis for an activity center meeting all design constraints is shown in Figure 7.1.

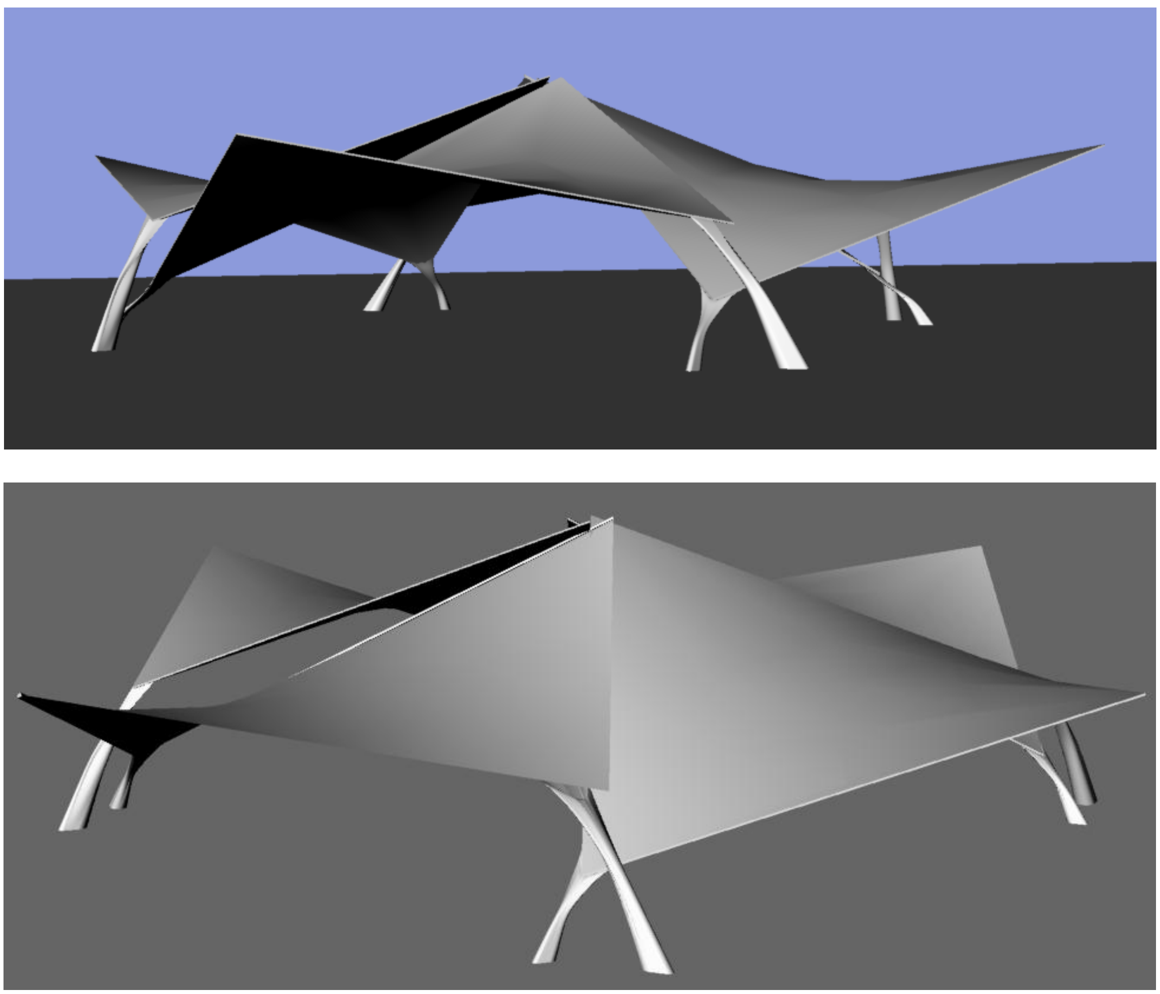


Figure 7.1: Renderings of the activity center conceptual design.

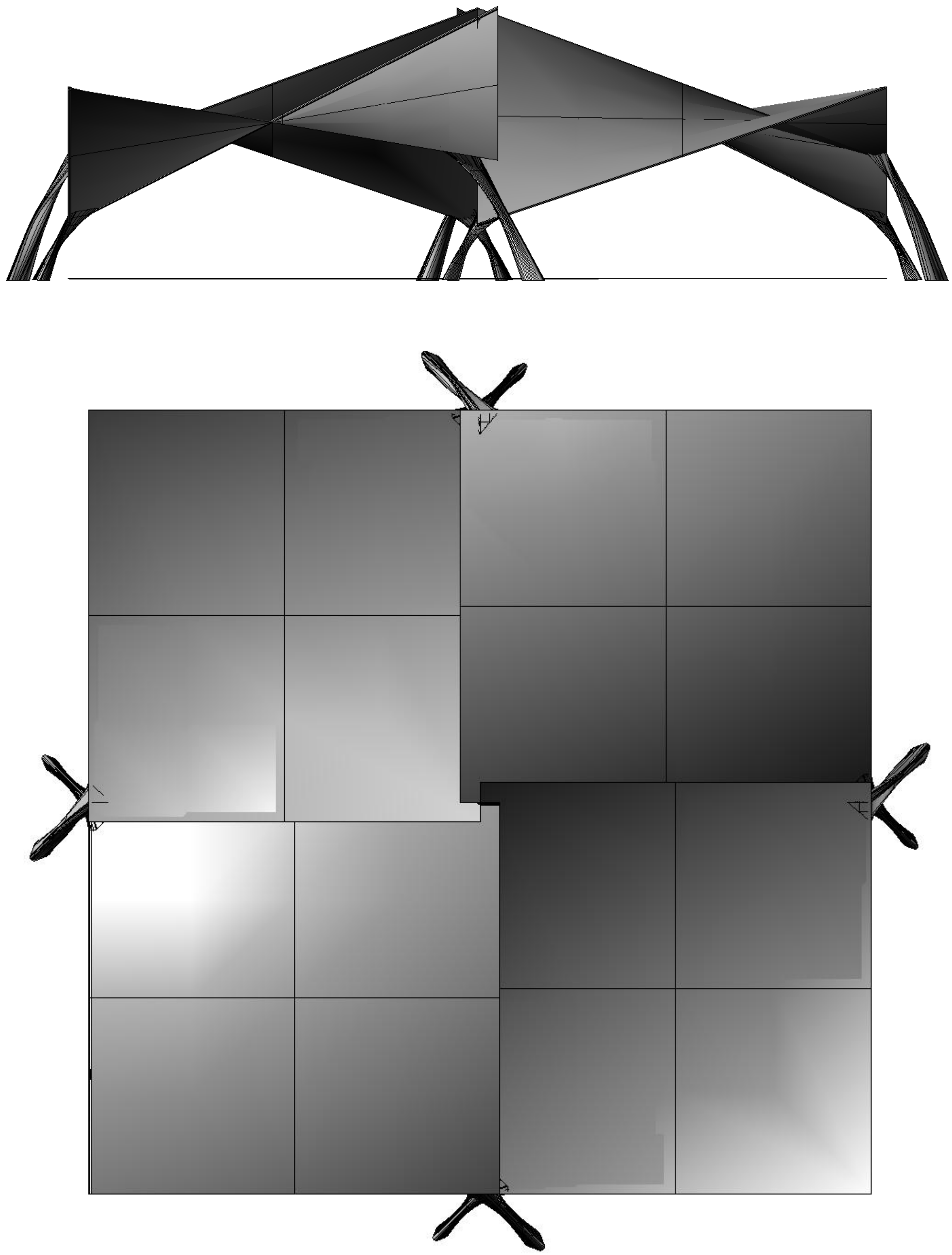


Figure 7.2: Top and side views of the activity center design concept.

The shell roof is a polar tessellation of titled double cantilever forms, arranged so that there are triangular gaps left between quadrants of the roof. These gaps are meant to allow indirect sunlight, which would enter the structure and be diffused by the underside of the adjacent shell. The effect would cut down on glare and the harmful effects of direct sunlight while at the same time allowing natural lighting into the structure. The roof is left open to allow easy access on all sides, channeling forces in the roof to only four areas of support. The open air configuration also transmits a breeze through the structure when available, cooling its exercising patrons. Despite the gaps and openings, the shelter will still protect from the rain, even guiding its flow over the roof to distinct capture points. At each of the gaps, there is an overhang on the upper hypar as well as a stiffening fold on the lower hypar which captures water flowing down the roof. Due to the slanting nature of the roof, most of the water will run off the structure at the supports, where it could either be collected or redistributed over surrounding areas. This prevents puddles from building up at the borders of the roof, which can hamper access and interfere with activities on rainy or recently rainy days.

Although the form is visually complicated, it is easily constructed using typical reinforced concrete methods. The formwork for the curved double hypar shell is made out of straight boards, the length of which is calculated by the straight line generators of the shape. Each double hypar quadrant is in theory structurally independent, enough so that the decentering process of one shell would not cause failure of an adjacent one due to an imbalance of forces. This means that the formwork could be reused, saving construction costs. Although there is some overlap between the shells, this would not be an insurmountable problem during construction, since there is only a one meter overhang compared to the 22.9 m wide shell. To build the structure, one shell would be poured and allowed to harden, after which the formwork would be removed and the scaffolding moved in place to the next shell. When the time comes to place the formwork on the next shell, it would have to be lifted vertically and then shifted over a few feet until it reaches the correct location and could be fastened in place. This procedure would be repeated until each shell has hardened, at which point the roof is complete. Before the formwork is removed from the shell, however, the supports must be in place. The curvature of the supports is designed to make them structurally efficient, but it also results in irregular shaping that is not easily formed by straight lines. The supports could still potentially be made cheaply through the use of precast elements or by using the earth as formwork and then hoisting them up vertically, if these procedures were developed.

The structure could be made entirely out of concrete and steel rebar, both of which are easily obtained in Latin American countries (WBCSD, 2005). The construction is materially efficient and labor intensive, involving the setting of rebar and the pouring of concrete. However, it is mostly low- tech, and could be completed by the members of a community that will house the structure. By combining material efficiency and ease of construction, the structure is also financially economical, which is a requirement for a structure in the context of a developing world community. In terms of the cultural sensitivity of the design, there is considerable precedent in Latin American architecture for the use of thin, curved concrete, especially in a breezy, open structure (Candela, 2011). This historical development of this style is discussed at length in Chapter 2. The architectural use of curved thin shells, indirect lighting, and smoothly plastic sculptural forms are staples of Latin American architecture, fitting in well with the region’s aesthetic sense (Kleinman, 2010). Considering Latin America’s architectural history, it is hoped that this conceptual design, visually complex as it is, would be accepted by a community and become a source of civic pride among residents, as was the case on a larger scale with the Miami Marine Stadium and other large hypar shell projects.

7.4. Form - Structural Analysis

Dimensions of the shell roof are given in Figure 7.3. Each shell is nearly square in plan at 22.9 meters by 24.0 meters, with the difference being due to the extra overhang of the high shell where they overlap along quadrant boundaries. The diagonal span between columns of the same shell is 32.3 meters, providing 2,090 square meters of free space underneath the roof in between the four sets of columns. The shell itself is 2 inches or 5.08 cm thick, which is smaller than the Miami Marine Stadium Roof, but slightly bigger than Félix Candela's standard thickness of 4 cm. Iterative analysis shows that stresses are not reduced with increased shell thickness, and the 5.08 cm thickness was based on a conservative estimate for the amount of concrete cover required for rebar in the shell. For comparison purposes and also because it is a viable option, the same grade of concrete used in the Stadium, 4,000 (psi), will be applied to the activity center design.

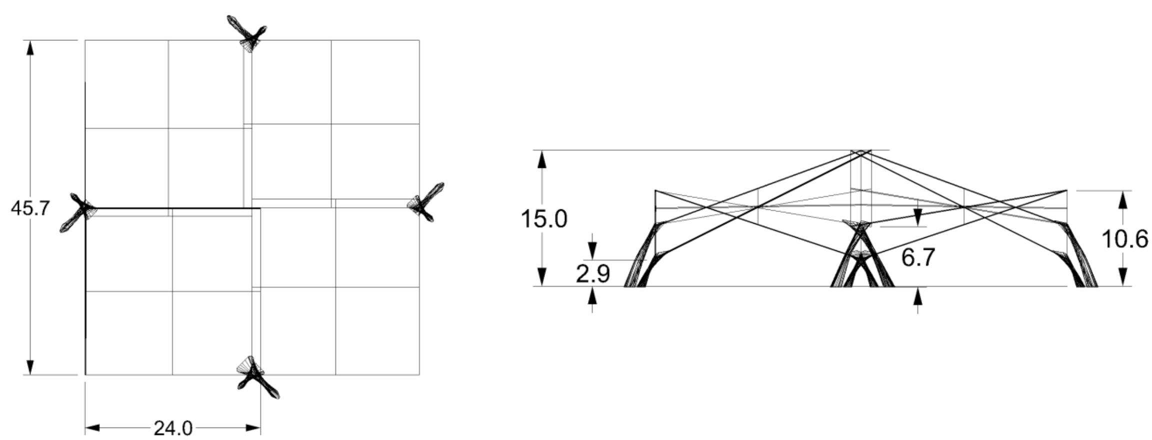


Figure 7.3: Overall dimensions of design concept in meters.

The general engineering design process for this hypar configuration began with a consideration of the force flow through one individual shell. It was expected that stresses throughout the interior of the shell would be very low, with the edges being in compression and larger compressive stresses building up at the two supports. This hypothesis was tested through an initial finite element model of the shape, using a method similar to the one used in the previous chapter. Due to the complexity of hand calculations for such a form, and also because it is only a conceptual model and gave intuitive results, only a finite element solution will be presented here. All load cases, element choices, and material properties used for this chapter are identical to those in the FEM analysis of the Miami Marine Stadium.

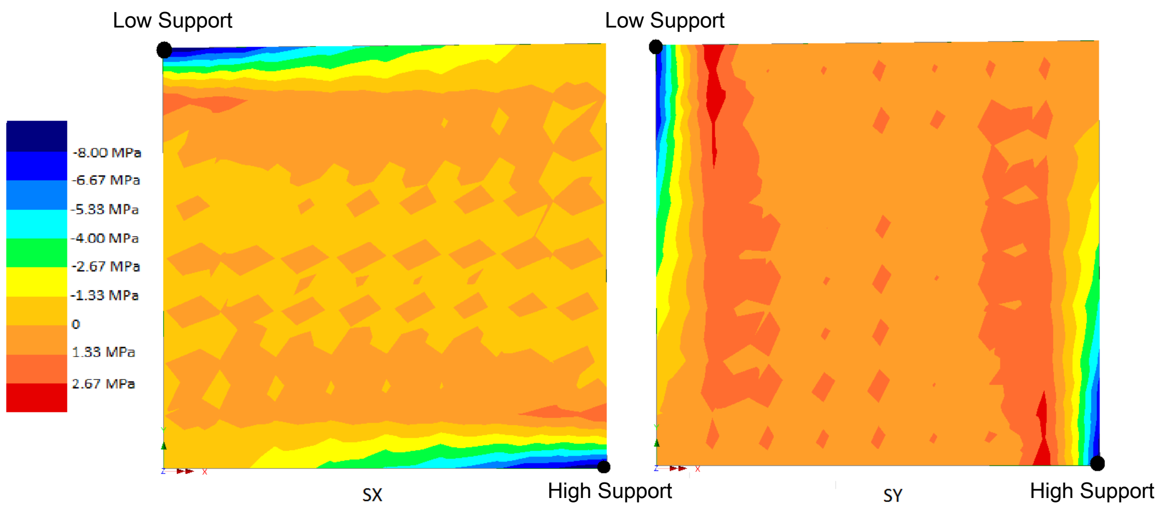


Figure 7.4: Element stresses in the X and Y directions, showing low magnitudes in the interior and compressive stresses along the edges towards the supports.

Figure 7.4 shows the element stresses in the middle of one double cantilever shell broken down into x and y components. As expected, the stress magnitudes remain small throughout most of the shell until they are near the supports. At the edges, the finite element model clearly shows compressive stresses in only the specified component direction of the plot, signifying longitudinal compression. In order to address this problem, stiffeners along the edges were added in such a way that they improve the structural performance of the roof and also add to the functionality of the structure—this will be discussed in more detail in the next section. Although these plots show the direction of forces flowing through the shell, they are not useful for directly comparing stresses to the strength of concrete. Due to the steep curvature of the structure, as well as the model's orientation on the *xy* plane, it is necessary to look at elemental stress instead to understand how *x*, *y*, and *z* component stresses are combining to produce total element stress.

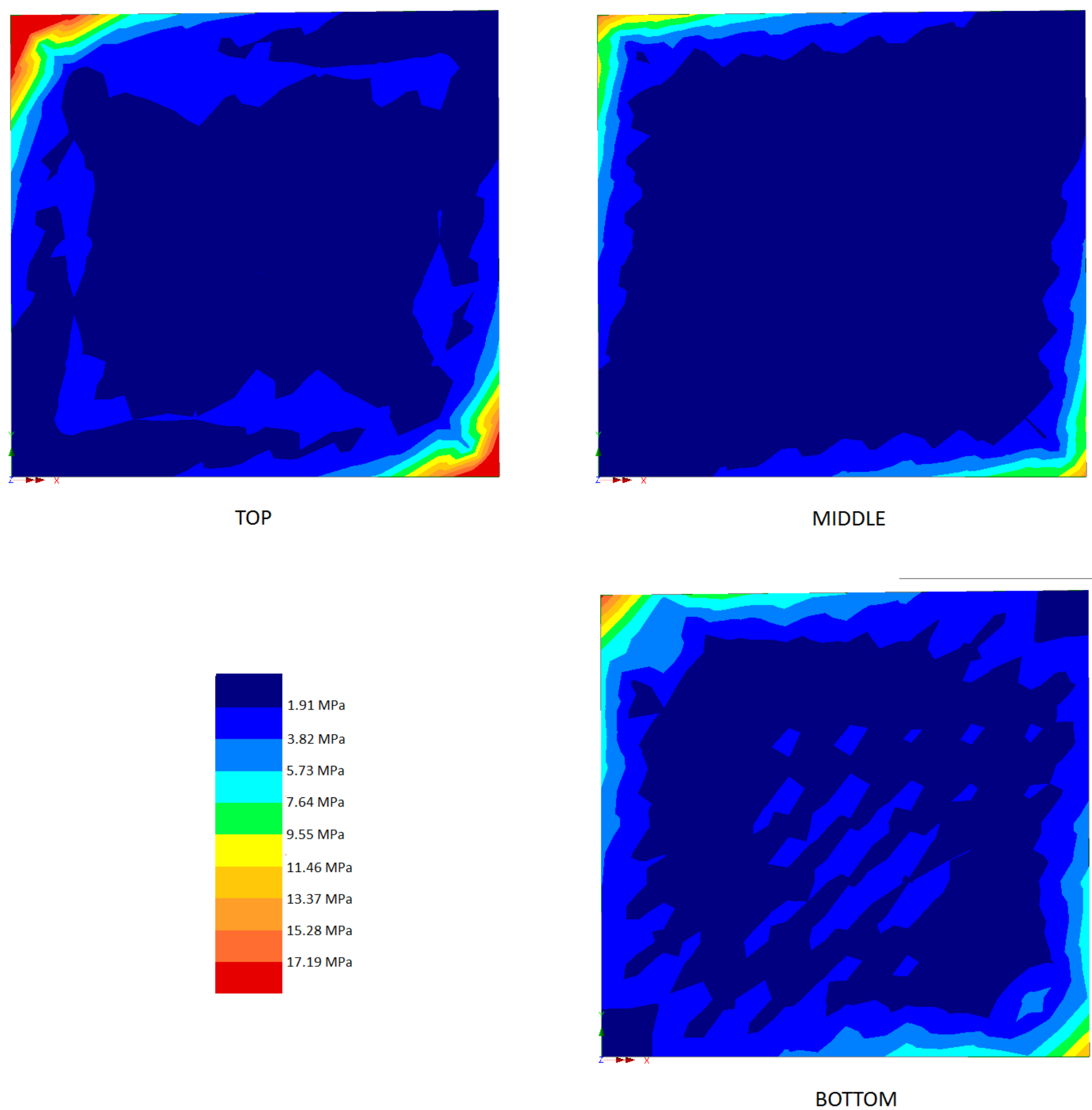


Figure 7.5: Magnitude of element stresses is one shell, throughout its cross section.

A plot of the stress magnitude acting on each element in the FEM model is given in Figure 7.5 for the top, middle, and bottom of the shell. The shell is in compression on the edges and near the supports, with the components of the x, y, and z compressive stresses adding to produce a resultant compressive stress on the elements in three dimensional space. Throughout most of the shell these stresses are less than 3.82 MPa, which is considerably lower than the reduced ultimate capacity of the concrete, specified as 13.1 MPa for this type. At the supports, the element stresses rise to unacceptable levels, but this is because the model contains a support located by a single point instead of a larger column connection. In the actual design, the column is tapered to meet the shell with a large cross sectional area, allowing for additional reinforcing and concrete to take the large connective stresses. The triangular shape of the tapered column where it meets the shell was determined from this stress pattern itself (See Figure 7.6). The integration of this taper into the rest of the column design will be discussed later. Excluding

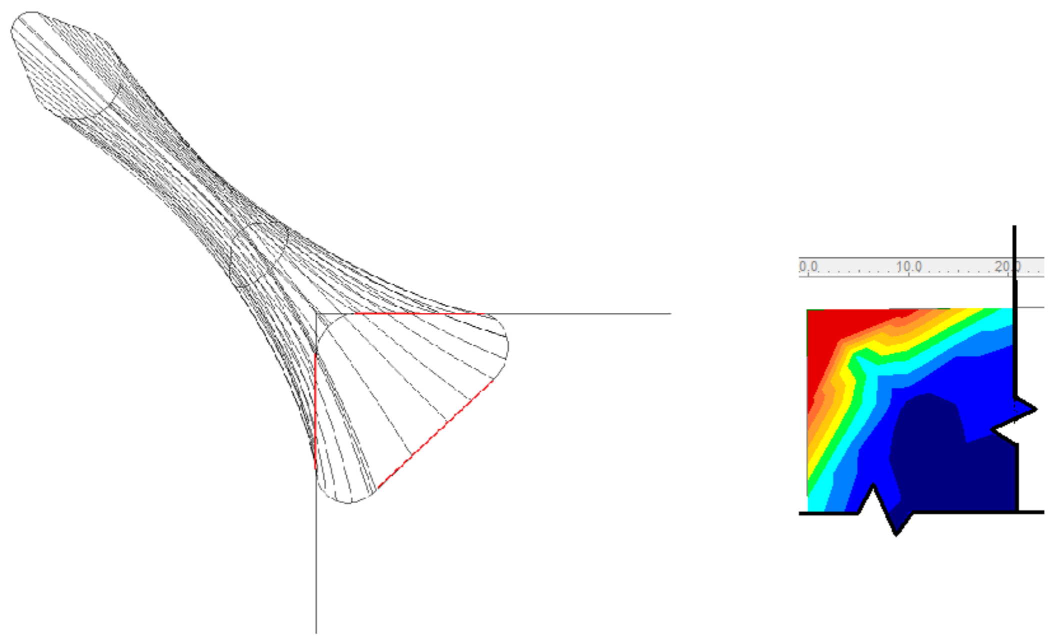


Figure 7.6: Cross section of the column where it meets the shell, a shape derived from the triangular stress pattern in the roof.

the special case of the shells near the supports, the low stresses in the rest of the shell under dead loads show the double cantilever hypar roof to be an efficient, viable design solution. Although other load cases would have to be applied in order to confirm this, the conceptual force flow and preliminary structural behavior have been validated by FEM analysis.

From the FEM model it was also possible to estimate reactions placed on the columns by the shell roof. These reactions contain a horizontal component which is larger in magnitude than the pure vertical component. The structural behavior of diagonal reactions makes vertical columns an inefficient choice, due to the large moments they would have to take compared to axial loads. In addition, although the higher shell support is directly above the lower support of the adjacent shell, it was decided that two separate columns would be the best use of material. After analyzing a number of shapes that integrated the support into one, combining columns acted to either force large local moments in section of the column between the two supports, or require a massive column larger than the scale required by the roof. To arrive at the shape and placement of two separate columns, the direction and magnitude of the reactions from the shell roof were considered separately for each support. Since the magnitudes of the *x* and *y* components were nearly equal at a given support, it was decided that the two columns should be slanted columns at a 45º angle out from the corner of their respective shells. In the preliminary design phase it was unknown if the tilt of the hypar would substantially unbalance the two supports, but the FEM model showed the reactions at the two supports to be roughly equal in magnitude.

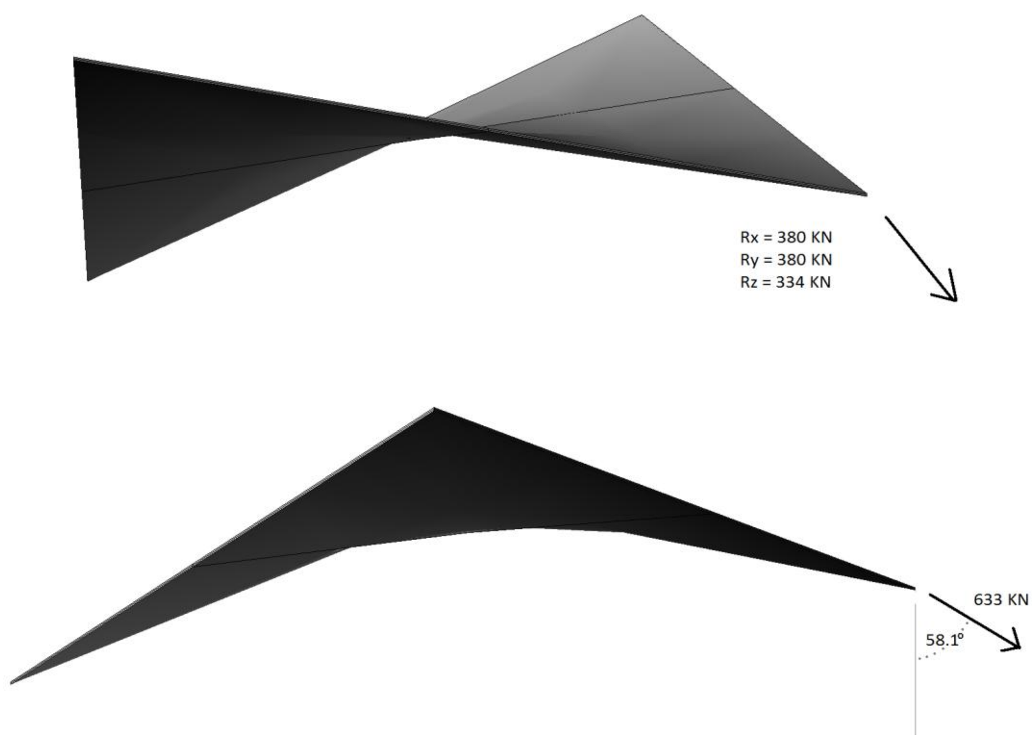


Figure 7.7: Reactions from shell roof.

Given the direction of the roof reaction, an angled column was developed to efficiently take both the horizontal and vertical components axially. For reasons of practicality, the column does not directly match the shallow angle of the reaction, since this would cause very long columns and require considerably more space outside the roof, which could be a concern if the shell is built beside another building. As such, the column must take at least some of the load in bending. To compensate for bending, the bottom of the column tapers to be wider in the plane of the reaction to take the increased moments placed upon it by the roof. In order to make sure the concrete was able to support both this axial load and moment, a preliminary P-M interaction diagram was constructed and consulted.

Taking all of these factors into consideration, the shape of the column was formed through a sweep of three distinct cross sections, following a curved line angled towards the shell roof (See Figure 7.8). The top cross section is increased and tilted to fortify the corner of the shell, the middle cross section of the column is sized based on the axial forces and medium-sized moment in the interior of the column, and the bottom cross section is given a longitudinally wider base to take the largest moment placed on the column. The column shown on the next page is for the lower support; the higher column uses the same logic, but is thicker through the middle to prevent against buckling.

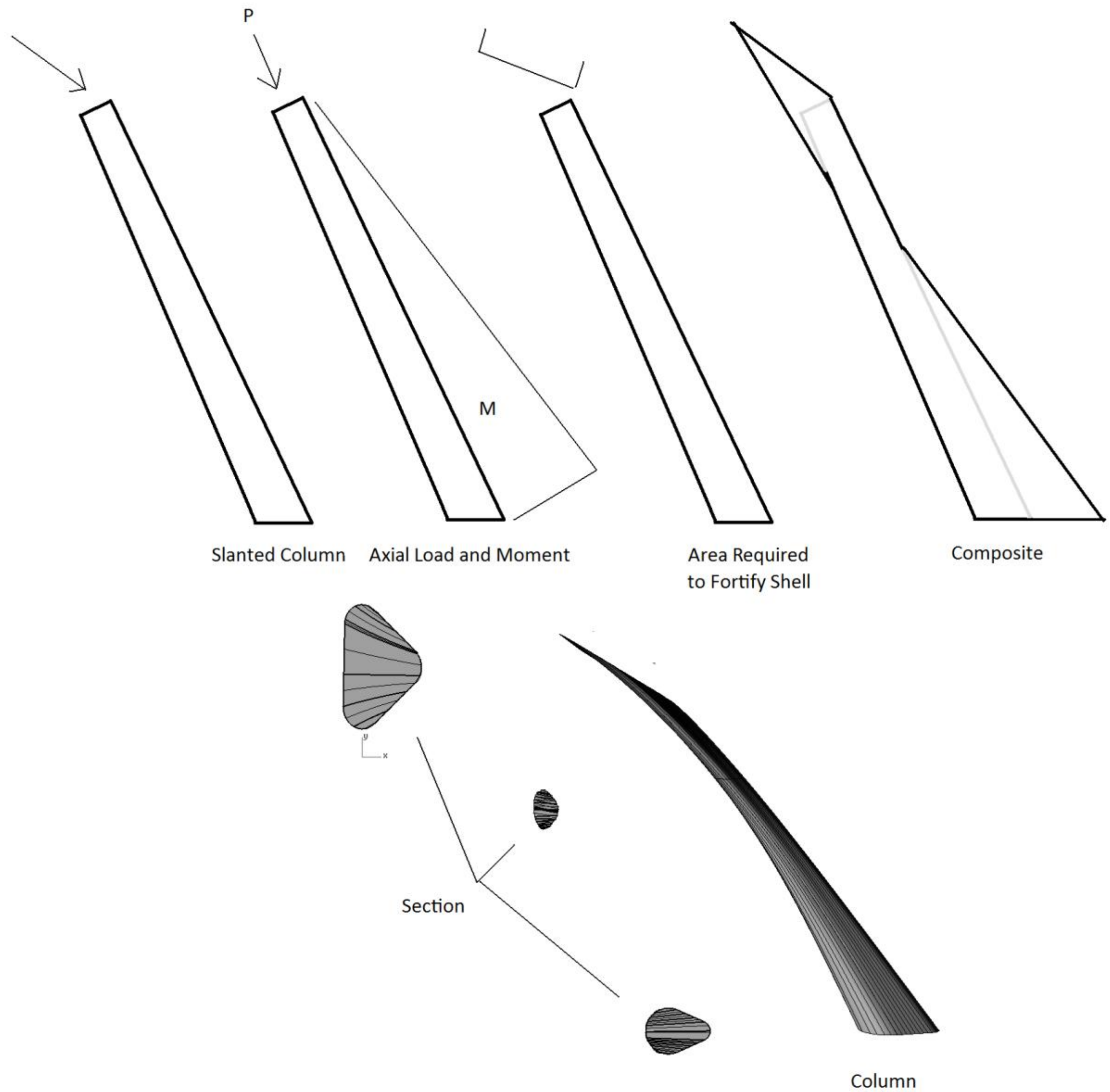


Figure 7.8: The design process for the shape of the column, integrating the individual needs of the shell connection, structurally efficient midsection, and large moment at the base to create a composite sweep.

Before being constructed, this conceptual design would require considerably more detailed analysis. Every structural element mentioned would have to be sized more precisely to maximize efficiency, and the design would have to be checked for local buckling and sheer. Additional load cases should also be evaluated in addition to the dead load of the structure, especially because asymmetric wind loads on the shell may be a worst-case scenario. A foundation plan would also have to be completed, and this poses some complications. In a normal double-cantilever design, the shell sits right on the ground, allowing for steel ties between the supports to help with horizontal reactions. A double cantilever also sometimes contains massive blocks of concrete at each support, even if steel ties are not possible. In any case, ties cannot be placed above ground since they would interfere with the activities contained by the structure, and the scale of the roof and slanting columns is such that they may not be feasible to place underground. Despite this need for further work, there is a logic to the conceptual design that uses an understanding of the hypar form to harness its inherent structural advantages. The shell and columns were configured and shaped specifically to perform their given structural tasks safely and efficiently, according to basic engineering principles. This intuitive approach to simplified analysis mimics established practices in thin shell design, giving confidence that the form will be improved but ultimately validated through further work.

7.5. Use - Functional Aspects

A discussion of the functional aspects of this shell design must begin with the programmatic configurations made possible by the size and shape of the roof. The activity center's purpose is a multiuse facility for a variety of desired sports and forms of exercise, and it has been scaled appropriately to a number of different sports. One possible configuration allows for a basketball court, indoor soccer court, and half-size indoor track to coexist, with the equipment of each sport being kept spatially from interfering with one other. Each of these sports would be afforded enough horizontal and vertical space to be played comfortably under the roof, although the central multipurpose space could only be used by one major sport at a time. Consider the diagrams showing the footprint of the building with each of these programs superimposed on top of it (See Figure 7.9). In this configuration, permanent or semi-permanent basketball hoops and soccer goals could be installed without interfering with any of the activity of the other sport, saving time in setting up a game. If these elements are moveable, then they can also be stored in one of the corners of the structure without interfering with the main central space. The track shown here is half the size of a regulation indoor track and would be unsuitable for high-level competition, but it could be adequate for casual joggers or walkers, encouraging an active lifestyle for those uninterested in higher intensity sports. To appeal to this demographic, all manner of fitness classes could be completed in the multipurpose space, and small exercise equipment such as pull-up bars or even weights, depending on the location of the structure and security measures taken to guard such equipment, could be included as well.

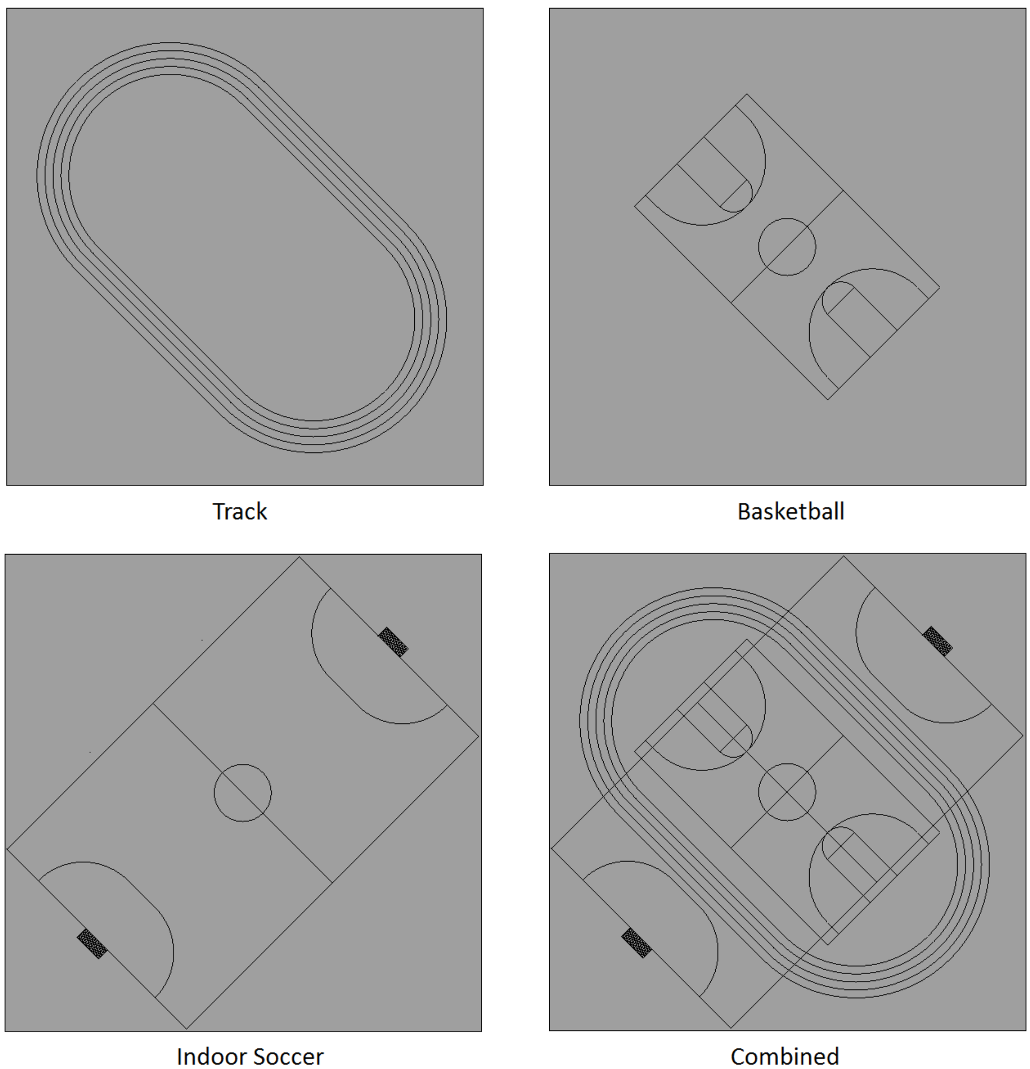


Figure 7.9: A possible programmatic configuration for combining different sports.

In addition to use in the strictest sense of the word, the shape of this structure promotes functionality by controlling the rainwater, lighting, and breezes of the site. In terms of rainwater, the double curvature of the roof acts as a natural gutter, using gravity to divert water to the lowest points on the roof. These low points occur at the supports, meaning that rainwater will tend to run off at four distinct locations instead of collecting in puddles all along the perimeter of the roof. To ensure that water does not run off into the gap in the interior, the stiffening edge of the lower interior hypar has been placed on top of the roof, which will act as a rain-stop and redirect

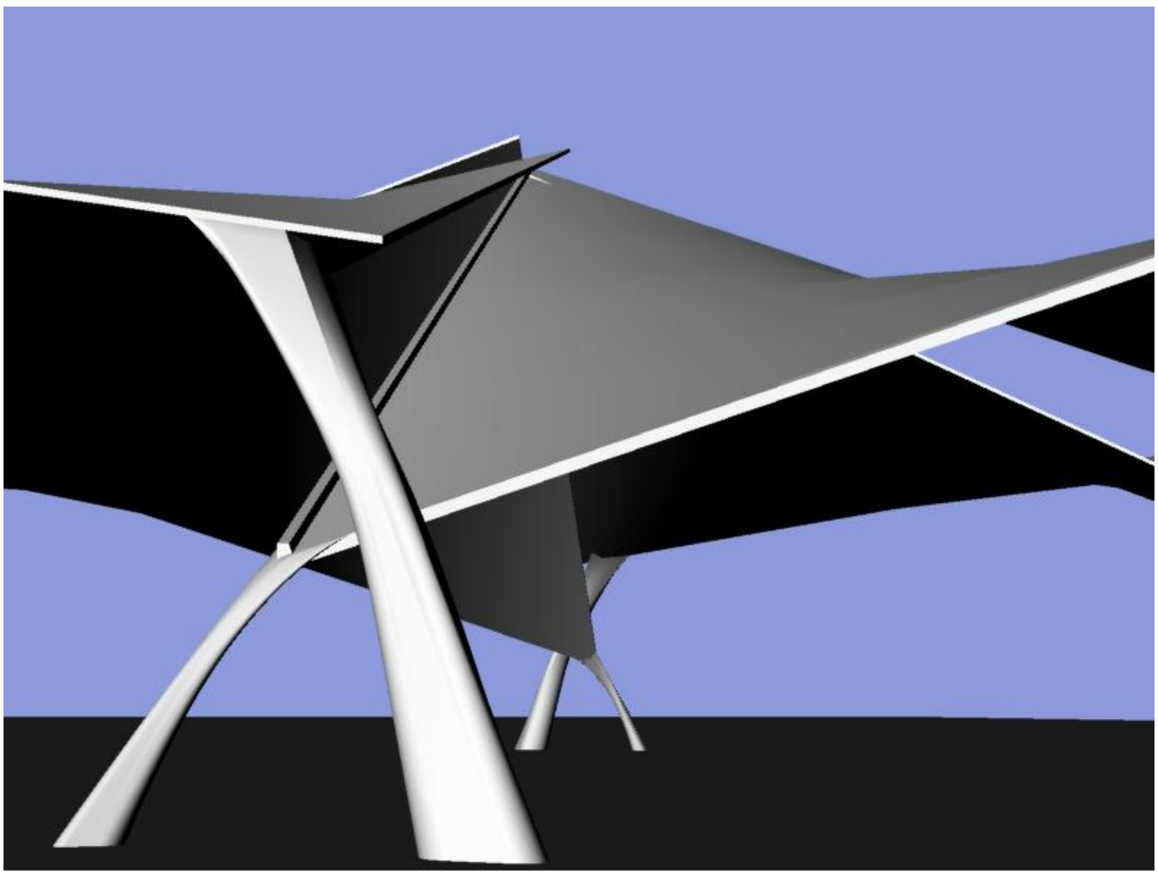


Figure 7.10: A rendered view of the structure showing the top lip on the interior shell, stopping rain from running off the roof inside the roof.

flow towards the column (See Figure 7.10). Since it is running to four distinct points, water could be collected in a basin for reuse, or it could be diverted into a storm sewer or redistributed back into the ground depending on location. The point of collection can be made even more precise by moving any of the outside stiffening edges to the top of the shell and leaving a small gap between perpendicular stiffeners or tapering the outside edge into the support. In areas with high rainfall, diversion and dispersion more be more desirable than collection, and careful planning must ensure that large rainfall episodes do not overload specific areas of the ground and cause flooding. However, this is outside the scope of a structural design.

The form of the structure also blocks direct sunlight during the hottest parts of the day, but allows indirect lighting through gaps in the roof protected by overhangs. The amount of light entering through this method can be adjusted by the color of the shell on both the interior and exterior. At some points during the day in which the sun is low, the gaps are large enough that direct sunlight will be able to reach portions of the interior of the stadium. However, given that both patrons and the spaces in between shadows will be moving around, their exposure to direct sunlight will be minimal. It is also worth noting that since the indoor soccer and basketball courts are oriented on the diagonal to the roof causing them to sit in between gaps, direct sunlight is not able to reach a basket or goal, meaning that players will never have to shoot into the sun. In case the sun or blowing rain becomes a nuisance, it would be simple to hang a tensile cloth sunshade from the inside of the shell over the gap, and this could be opened or closed depending on weather.

In each case, the functionality of the structure is enhanced through the geometric properties of the hypar form. The vast vaulted space that can be efficiently spanned by a hypar shell allows for a variety of programmatic uses for the structure. Water is directed by the double curvature towards the supports, and a tessellation of tilted hypars allows for lighting to enter the underside of a shell and be gently diffused by a curved interior with no sharp edges to block the light. The only exception may be breeze, because any open structure would allow for breeze to enter and cool an interior. Nevertheless, the hypar form indirectly improves the ability of a structure to be open to the air by allowing for structurally efficient designs with free edges.

Through these design aspects, the activity center structure illustrates an example of the synergistic relationship between 'use' and hypar shells.

7.6. Sustainability – Environmental, Economic, and Social Considerations

It has been argued previously in this thesis that the two most important components of sustainability as it relates to the structural engineering of thin shells are efficiency of materials and durability. Although other factors involving sustainability certainly apply to structures, beyond the realm of these two metrics it becomes difficult to quantify the impact of good structural design. This is largely due to the specific nature of sustainability metrics such as LEED or SBAT (Sustainable Building Assessment Tool©), which include considerations as wide-reaching as parking, proximity to schools and banks, efficiency of lighting fixtures, and access to the internet and health information (Sebake *et. al.*, 2008). Furthermore, these metrics are meant to evaluate buildings already in use that have accurate, reliable information concerning the building's construction, occupancy, and performance. This makes the application of holistic sustainability systems difficult in the case of a conceptual structural design. However, SBAT was developed in Pretoria with the specific intent of evaluating a building's impact on sustainable development while considering the environment, economy, and social structure of a developing nation (Gibberd, 2001). This objective makes SBAT directly relevant to the activity center's location as well as its vision for sustainability as defined by this thesis. While preparing for the FIFA World Cup hosted by South Africa in 2010, an alternative SBAT metric was also established for evaluating stadiums, adjusting criteria to more accurately reflect the usage of a stadium (Sebake, 2008). Although the exercise structure proposed here is not meant to be a stadium that accommodates large crowds, the adjusted *SBAT for Stadia* more closely matches its context and usage than the generalized SBAT system for commercial buildings. Thus, the SBAT could prove useful in the case of design, even if its categories cannot be calculated accurately.

To get the most value out of a discussion of sustainability for this design, the efficiency of materials and durability of the shell will first be addressed directly. Following these two topics, *SBAT for Stadia* will be qualitatively applied to the conceptual activity center design. Since many criteria are still not applicable or impossible to quantify without a construction bid, the loose application of SBAT will consider a reasonable potential ceiling for SBAT scores given the shell's design. Through this joint approach of direct calculation and loose application, it is hoped that an accurate depiction of this design's sustainability will be reached.

As previously shown, the structural efficiency of a concrete structure directly affects its environmental sustainability, since concrete has a considerable amount of embodied energy. The Structural Efficiency Impact Ratio for this conceptual design is 44.2 pounds CO_2 emissions per square meter, ranking it fifth best out of seven if considered with Candela's hypar shells modeled in Chapter 4. A substantial contributor to the amount of material per area is the columns, which could be made more efficient through further iteration and are much taller than columns in Candela structures as required by its architectural program. Despite the bulky aboveground supports, the activity center still ranks among Candela's best structures in efficiency, performing much better than a traditional concrete design.

Unlike structural efficiency, a shell's durability is mostly a function of the selection and preparation of materials used in the structure, especially given the Latin American climate of the activity center. The durability of concrete is defined as "its ability to withstand the damaging effects of environmental factors and to perform satisfactorily under service conditions" (Soroka, 1993, p. 179). According to Soroka's *Concrete in Hot Environments*, hot-weather conditions can complicate durability both directly through being an aggressive environment, and also by making it difficult for concrete to reach a high quality during mixing. As a general rule, high concrete porosity and cracking allow chemical corrosion of concrete and reinforcing, reducing the service life of a structure. Thus, it is important to make sure concrete has a low permeability when constructed to main durability and in effect sustainability, although these properties must be balanced with the structural demands of a concrete type (Soroka, 1993). Although well-mixed, accurate concrete is difficult to create in a low-tech, developing world construction context, it is entirely possible if made a priority during the building phase. The use of galvanized steel could also help durability, as long as this material is easily available.

Following the direct discussion of structural efficiency and durability, this thesis will now look at the three types of SBAT criteria as they relate to the activity center. For quick reference, the breakdown of each category will be given in the text; to understand the specifics of these

		Target
EC 1	**Local economy**	
EC 1.1	Local labour force	80%
EC 1.2	Local building materials	80%
EC 1.3	Local components and equipment	80%
EC 1.4	Local furniture and fittings	80%
EC 1.5	Maintenance	40%
EC 2	**Efficiency**	
EC 2.1	Capacity	60%
EC 2.2	Occupancy	60%
EC 2.3	Space per spectator	1.05
EC 2.4	Shared parking	40%
EC 2.5	Multiple use	100%
EC 3	**Adaptability**	
EC 3.1	Alternative uses	67%
EC 3.2	External space	80%
EC 3.3	Services	80%
EC 3.4	Media flexibility	100%
EC 3.5	Suite flexibility	100%
EC 4	**Ongoing costs**	
EC 4.1	Water consumption	4.0
EC4.2	Energy consumption	4.0
EC 4.3	Cost centres	80%
EC4.4	Maintenance & cleaning	100%
EC4.5	Facilities management	100%
EC 5	**Capital Costs**	
EC 5.1	Training	1%
EC5.2	Labour intensity	2.4
EC 5.3	SMME support	40%
EC5.4	Sustainable technology	3%
EC 5.5	Private sector funding	2%

Figure 7.11: Economic criteria for SBAT evaluation. Graphic courtesy of *SBAT for Stadia*.

criteria, it is necessary to consult the *SBAT for Stadia* guide directly. The economic evaluation criteria for the SBAT adjusted for stadiums is given in Figure 7.11. In the design for the activity center, there is reason to believe that the design could achieve high scores in both local economy and adaptability, directly as each are written in the criteria (*SBAT for Stadia,* 2010). The construction process would require considerable amounts of local labor for the building formwork and pouring concrete. These jobs would be low-tech, and given the prevalence of concrete construction in Latin America, it is reasonable to assume that the materials could be locally obtained, with labor and maintenance contracts going to local contractors employing members of the community. In terms of adaptability, the open, non-partitioned nature of the activity center design allows for the easy use of any program that requires a large space. Although the purpose of the structure is to provide room for exercise, it could just as easily be used as a center for community meetings, plays, celebrations, religious services, or any other such event. If managed by a competent NGO or community committee, and if it is shared by local entities such as nearby schools or churches, it could achieve a high level of efficiency and economic viability. Capital costs would be low due to economic construction, and ongoing costs could be minimal and restricted to maintenance and management, especially if the open air activity center does not include water and additional lighting. Even if these elements were integrated into a later design, there is potential for efficiency given the ability of the roof to control rainfall and natural lighting. A number of the criteria such as media, suites, and parking density do not apply, since the structure is not designed for huge crowds to drive to it for events. However, despite the differences between an activity center and a large, full-service stadium for which the metric was developed, these economic criteria show how this design concept could be economically sustainable if managed correctly.

A number of the *SBAT for Stadia* environmental criteria also apply directly to the activity center roof design. The targets for these environmental sustainability metrics are given in Figure 7.12.

		Target
EN 1	**Water**	
EN 1.1	Rainwater	100%
EN 1.2	Water Efficiency	100%
EN 1.3	Runoff	50%
EN 1.4	Grey water	100%
EN 1.5	Planting	100%
EN 2	**Energy**	
EN 2.1	Location	80%
EN 2.2	Passive environmental control	100%
EN 2.3	Energy efficiency	0.145
EN 2.4	Control and BMS	100%
EN 2.5	Renewable energy	20%
EN 3	**Waste**	
EN 3.1	Waste management facilities	100%
EN 3.2	Waste minimisation - front house	100%
EN 3.3	Waste minimisation - back house	100%
EN 3.4	Demolition	100%
EN 3.5	Construction waste	90%
EN 4	**Site**	
EN 4.1	Brownfield site	100%
EN 4.2	Neighbouring buildings	100%
EN 4.3	Vegetation	50%
EN 4.4	Construction process	100%
EN 4.5	Landscape inputs	75%
EN 5	**Materials & Components**	
EN 5.1	Roof	10%
EN 5.2	Concrete	50%
EN 5.3	Roof efficiency	139
EN 5.4	Superstructure efficiency	4.3
EN 5.5	Hazardous materials	100%

Figure 7.12: Environmental criteria for SBAT evaluation. Graphic courtesy of *SBAT for Stadia*.

To begin with, the Material & Components section can be calculated as early as the conceptual design phase. For example, the roof efficiency for the activity center is 110 kg/m^2, or 20% better than the optimal target rate of 139 kg/m^2 given by *SBAT for Stadia*, providing further evidence of the efficiency of shells. The rest of the subcategory refers to the amount of recycled materials involved in construction, which could reasonably match the target in this design depending on supplier. The subcategory of energy could also contain high scores in the relevant categories, given the design's open-air layout with passive control of environmental conditions. In the water subcategory, the design would score only medium high in the measurements that apply. Although it allows for the possibility of rainwater harvesting, the roof contains no absorbent material, which would cause considerable runoff if uncollected. The subcategory of waste is in a similar situation of medium performance, since the only two relevant metrics are demolition and construction waste. Although a concrete structure is easily demolished, much of the formwork must be discarded after construction. The activity center's score would be enhanced considerably if these materials are recycled after use. The category of site is outside the scope of this design concept, since a specific location has not been proposed in order to highlight the versatility of the design. However, there is nothing preventing the design from achieving high marks in this category, especially if constructed in an urban environment or on another brownfield site.

The third *SBAT for Stadia* criteria consider the social aspects of a building's construction and use, and it can be used to evaluate the social sustainability of the design within the context of its community (See Figure 7.13). Again, many of these metrics do not apply to a conceptual design, but instead would have to be evaluated for a specific site. However, the two relevant subcategories of Spectator Comfort and Participation & Control include criteria in which the structure would score well. In an open-air roof, the shading and ventilation would provide protection from the sun while allowing as much breeze as possible, resulting in a high degree of unconditioned patron comfort. For the category of Participation & Control, the design could only be successful in a context that effectively addresses these aspects of the community center before, during, and after construction. The Participation & Control criteria refer to an efficient management structure consisting of a partnership between local entities, the provision of social space, local access, and regular community input into the life of the structure. All of these are included in the expressed purpose of the design. In addition, it is hoped that the elegant

		Target
SO 1	**Spectator Comfort**	
SO 1.1	Shading	100%
SO 1.2	Ventilation	100%
SO 1.3	Large screens	2
SO 1.4	Crowding	1.500
SO 1.5	Proximity	100%
SO 2	**Inclusive Environments**	
SO 2.1	Transport	400
SO 2.2	Wayfinding	1
SO 2.3	Space	100%
SO 2.4	Toilets	100%
SO 2.5	Distribution	100%
SO 3	**Access to Facilities**	
SO 3.1	Accommodation	1
SO 3.2	Banking	0
SO 3.3	Pedestrian / cycle routes	20%
SO 3.4	Food	100%
SO 3.5	Drink	100%
SO 4	**Participation & Control**	
SO 4.1	Environmental control	100%
SO 4.2	Role players	100%
SO 4.3	Social spaces	100%
SO 4.4	Shared access	100%
SO 4.5	Local community	100%
SO 5	**Education, Health & Safety**	
SO 5.1	Education	100%
SO 5.2	Website	100%
SO 5.3	Health	100%
SO 5.4	Health & Safety	100%
SO 5.5	Security	100%

Figure 7.13: Social criteria for SBAT evaluation. Graphic courtesy of *SBAT for Stadia*.

appearance of the structure would help to make the activity center a popular central space in the community as well as a source of civic pride. Furthermore, the structure is designed to promote healthy lifestyles in response to the potential health risks posed by inactivity, which can improve the quality of life in a community. Ultimately, it is hoped that these factors would work to elevate community interest, leading to a high score in the category of social sustainability.

In the areas of economic, environmental, and social sustainability, the activity center design concept has proven to have the potential for high scores on a number of different sustainability metrics. Many of these high scores depend on the specifics of site selection, construction, and community management, details which will be completed as further work. Still other established metrics do not directly relate to such an open-air structure, and these must be ignored. Aside from these two types of criteria, there are many metrics of sustainability which depend directly on the intrinsic properties of the conceptual hypar shell design, and in these metrics the activity center would score well.

7.7. Synergy of Design Aspects and Further Work

In many different areas of this conceptual design for an activity center in Latin America, the interconnected drives of form, use, and sustainability work together to create the potential for a successful structure. For example, the form's structurally efficient design leads to both a reduced energy footprint as well as a lower cost. The versatility of architectural uses ensures accessibility for the entire community, making it more socially sustainable. On the edges of shells, stiffening ribs act to both take the loads of the roof as well as to divert rainwater from flowing into the central space. These examples of synergy between the three different governing forces are a direct result of making them the focal point of the design in its earliest phases, contributing to a better structure overall.

In order to make this design viable, considerably more work would have to be completed, both in terms of engineering and community development planning. On the engineering side, the structure would need additional, more thorough analysis than the simplified version presented in this chapter. The design process would have to consider additional load cases such as wind and other live loads. A more nuanced analysis would use an iterative process to make the shell and columns even more efficient than those initially proposed. In addition, the foundation and reinforcing plan would have to be drawn up, taking into account the forces that will be flowing through the roof. A detailed construction procedure would also have to be established. Although the procedure would be based on the principle of reusing formwork among shells, there are considerably more ways in which well-planned construction techniques can lead to lower cost and less environmental waste. Nevertheless, even though a vast amount of design work still remains to make this a tangible structural solution, the design logic dependent on an understanding of the interplay between form, use, and sustainability will carry through the rest of the design process.

8. Conclusions

As the fields of structural engineering and architecture move forward and look to solve design problems posed by the twenty-first century, they can look to past successes of the hypar shell for inspiration. For these historical precedents, contemporary designers can thank the innovative master builders of various shell schools, who applied the different approaches of mathematics, visual aesthetics, and simple construction to leave a vast legacy of shell examples. Although the application of hypar shells is in some sense limited by their distinctive characteristics of thinness and curvature, there are numerous architectural programs and structural demands for which hypar shells are a practical, sustainable design solution. Throughout the study and classification of the form, hypar shells have been shown to be spatially versatile, constructed efficiency, and to behave intuitively. Within the many examples presented in this thesis, there are a number of distinct hypar configurations and design trends that arose in response to a particular set of constraints, and future designers facing similar applications can use the system as a reference for what has worked before. With recent technological and analytical developments in the shell building community as well as the current design climate of sustainable design, information concerning shells may become even more relevant in the future.

The case of the Miami Marine Stadium presents a specific example in which a hypar shell was successfully employed in an American urban context, adding value to the conversation concerning shells. The stadium design represents a compromise between a visionary architect and a highly skilled engineer, who worked together to produce a structural marvel that is well loved in the Miami community. The combination of form and scale in the stadium was largely unprecedented, and the lack of available analytical tools caused the engineer to propose a hybrid structural system in the roof consisting of both hypar umbrellas and elements of folded plate design that were popular at the time. This thesis praises the engineer for creating a daring and creative, yet pragmatic design that has stood strong though hurricanes and minimal maintenance, a testament to its soundness. At the same time, finite element models show the original stadium could have performed just as well with a considerable material reduction had the folded plate stiffeners been removed, further evidence of the inherent efficiency of hypar forms. The stadium has also shown to be versatile in use and comfortable for patrons, while its longevity and restoration movement have proven its durability and social sustainability in the city of Miami.

The conceptual activity center design aims to replicate the previous successes by the stadium and other shells through the application of a new shell configuration to a specific region and social issue. The design attempts to be holistic in its approach and achieve a synergy between form, use, and sustainability that is necessary for a hypar shell to effectively address the needs to which it responds. For the conceptual design to be realized, more work would have to be done concerning additional load cases and connection, foundation, and construction details. In the same vein, more research could be completed on additional possible hypar configurations, since many good examples exist outside of this thesis. Such research into historical shells would supplement the advances currently being made by those who are focusing on advanced construction techniques, materials science, and numerical modeling in an attempt to make shells more viable in the future. As illustrated through both the historical synthesis and quantitative analysis in this thesis, establishing the further viability of thin shells is a worthwhile endeavor. In certain applications, hypar shells have exhibited beauty of form, adaptability of use, and environmental and social sustainability, making them a good choice for structural additions to the built environment.

9. References

Ademiluyi, I. A., 2010. ‘Public Housing Delivery Strategies in Nigeria: A historical perspective of policies and programmes’. *Journal of Sustainable Development in Africa*, 12(6).

Adriaenssens S., Lowinger R., Hernandez J.,Brown N., Halpern A., Aye Z .M., Prier M., 2012. ‘The Shells of the Miami Marine Stadium: Synergy between form, force and energy’. *IASS-IACM 2012: 7th International Conference on Computational Mechanics of Spatial Structures, Sarajevo, Bosnia and Herzegovina.*

Adriaenssens, S., Brown, N., Lowinger, R., & Hernandez, J, 2012. Structural analysis of reinforced concrete folded hyperbolic paraboloid: a case study of the Miami Marine Stadium. *International Journal of Architectural Heritage*: under review.

Allen E. & Iano, J. 2012. *The Architect Studio’s Companion: Rules of Thumb for Preliminary Design*. Hoboken, NJ: John Wiley and Sons, Inc.

Antoniades, Christina, 2010. ‘In their lives: A revolution interrupted this architect’s rise, but he never looked back’, *Washington Post Magazine*, 2010.

Austin, Tom, 2011. ‘Other Cities Have Stadiums, but We Have A Marine Stadium’, *Ocean Drive Magazine*, September/October.

Baker, Kovalevsky, & Rish, 1972. *Structural Analysis of Shells*. New York: McGraw-Hill.

Battelle, 2002. ‘Toward a Sustainable Cement Industry: Summary Report’. *World Business Council for Sustainable Development.* Available at: www.wbcsdcement.org

Beles, Aurel & Soare, Mircea V, 1976. *Elliptic and Hyperbolic Paraboloidal Shells Used in Constructions*. London: Editura Academiei Romane.

Billington, David P., 1982. *Thin Shell Concrete Structures*. 2nd ed. McGraw-Hill.

Billington, David P., 1983. *The Tower and The Bridge*. Princeton University Press.

Borer, Ted, 2011. ‘On Sustainable Thinking”, Presentation to CEE 477: Engineering Design for Sustainable Development’. Princeton University: November.

Brainerd, Michael L., Tumialan, J. Gustavo, & Bronski, Matthew B., 2011. “Evaluating Current Conditions of Miami Marine Stadium”. *Concrete International Magazine*, February.

Bull, Fiona C., Armstrong, Timothy P. , Dixon, Tracy, Ham, Sandra, Neiman, Andrea, & Michael Pratt, 2004. ‘Physical Inactivity’ in *Comparative Quantification of Health Risks: Global and Regional Burden of Disease Attributable to Selected Major Risk Factors.* World Health Organization: Geneva, Switzerland.

Candela, Félix, 1960. ‘General Formulas for Membrane Stresses in Hyperbolic Paraboloid Shells’. *Journal of the American Concrete Institute.* 32(4). October: 335-71.

Candela, Félix, 1964. ‘La obra de Pier Luigi Nervi y su influencia en la arquitectura contemporánea’, *Cuadernos de arquitectura.* 15. November: XXVI.

Candela, Félix, 1958. ‘Understanding the Hyperbolic Paraboloid: Part 2, Stress Analysis for any Hyperbolic Paraboloid’. *Architectural Record*. August: 205-207.

Candela, Hilario, 2011. Private communication with S. Adriaenssens.

Clark, Nanette, 2009. ‘Félix Candela Thin-Shells for October 20th’. *An Engineer’s Aspect*. [Online]. Available at: anengineersaspect.blogspot.com

Computers and Structures, Inc., 2004. *CSI Analysis Reference Manual*, Berkeley, California: September.

Conniff, Richard, 2005. ‘Counting Carbons’, *Discover*. August.

City of Miami. Website [online]. Available at: www.miamigov.com

Dischinger, Franz, 1928. ‘Schalen und Rippenkuppeln’, *Handbuch fur Eisenbetonbar.* 3rd Ed. Berlin: 151-371.

Domingo, A., Lazaro, C, & P. Serna. ‘Design of a Thin Shell Steel Fibre Reinforced Concrete Hypar Roof’. *Universidad Politecnica de Valencia*. [Online]. Available at: http://www.cmdingenieros.com/articulos/1999_Domingo_Lazaro_Serna_DesignOfAThinShellSteelFiberReinforcedConcreteHyparRoof.pdf

Dorf, Richard, 1996. *Engineering Handbook*, New York: CRC Press.

Draper, Powell, 2006. *Félix Candela and the Chapel of Lomas de Cuernavaca: How a Master Builder Employed the Hyperbolic Paraboloid For a Thin Shell Concrete Structure.* Princeton University Master’s Thesis.

Draper, Powell, 2008. *Building for the Future: Evaluating the Current Viability of Thin Shell Structures*. Princeton University Ph.D. Dissertation.

Dwell. ‘Rethinking Preservation: Entrance to the Sacramento zoo’. [Online]. Available at: www.dwell.com/contests/rethinking-preservation/submissions/entrance-to-the-sacramento-zoo

Friends of Miami Marine Stadium. Website [Online]. Available at: www.marinestadium.org

Garlock, Maria & Billington, David P., 2008. *Félix Candela*: *Engineer, Builder, Structural Artist*. New Haven: Yale University Press.

Giedion, Sigfried, 1941. *Space, Time and Architecture*. Harvard University Press.

Gibberd, J., 2001. ‘Building Sustainability: How Buildings can support Sustainability in Developing Countries’. Continental Shift 2001 - IFI International Conference. Johannesburg.

Guardado Díaz, Julio José, 2010. *Estudio de la estructura del Jacaranda Nightclub de Félix Candela*, Escuela técnica Superior de Arquitectura de A Coruña.

Guthold, Regina, Ono, Tomoko, Strong, Kathleen L., Chatterji, Somnath, & Alfredo Morabia, 2008. ‘Worldwide Variability in Physical Inactivity: A 51- Country Survey’. *American Journal of Preventative Medicine,* 34(6). June: 486-94.

“Havana's 20th century architectural gems”. *Building Design,* 2006. 9(9).

“Interview with Jack Meyer, Engineer of the Miami Marine Stadium”. *Friends of Miami Marine Stadium*. 12 May 2010. [Online]. Available at: http://www.marinestadium.org/index.php/news/445

Jones, Jenny, 2010. ‘Miami Marine Stadium Undergoes Structural Analysis’. *Civil Engineering*. October.

Jordon, Rob, 2001. ‘Preserving the Miami Marine Stadium’. *Dwell*. [Online]. Available at: www.dwell.com/articles/preserving-the-miami-marine-stadium.html

Ketchum, M. & Ketchum, M.,1997. *Types and Forms of Shell Structures: An Ideabook for Designers.* [Online]. Available at: www.ketchum.org/ShellTandF/index.html

Kleinman, Rebecca, 2010. ‘Brute Force: A preservation battle revives interest in the career of architect Hilario Candela’. *Modern Magazine*. Fall.

Kibert, Charles J, 2005. *Sustainable Construction: Green Building Design and Delivery*, Hoboken, NJ: John Wiley and Sons.

Larson, Roland, Hostetler, Robert, & Edwards, Bruce H., 1994. *Calculus with Analytic Geometry*. 5th Ed. Lexington, Massachussets: D.C. Heath and Company.

LEED 2009 for New Construction and Major Renovations, U.S. Green Building Council, Inc.

Lozano-Galant, Jose Antonio & Ignacio Paya-Zaforteza, 2011. ‘Structural Analysis of Eduardo Torroja’s Fronton de Recoletos’ Roof’. *Engineering Structures*. 13 January.

MacIntosh, Heather, 2000. ‘Kingdome: The Controversial Birth of a Seattle Icon’. *HistoryLink.org.* [Online]. Available at: www.historylink.org/index.cfm?DisplayPage=output.cfm&file_id=2164

McCormac, Jack C., & Brown, Russell H., 2009. *Design of Reinforced Concrete*. Hoboken, NJ: John Wiley & Sons, Inc.

Melnick, Jordan, 2012. ‘The Death (and Life?) of Miami’s Marine Stadium’. *Atlantic Magazine-Place Matters*. 8 March.

Meyer, Christian, 1996. *Design of Concrete Structures*. Upper Saddle River, NJ: Prentice Hall.

Meyer, Christian & Sheer, Michael, 2005. 'Do Concrete Shells Deserve another Look?'. *Concrete International*, October.

Meyer, Jack., 2012. [Interview] (February 2012).

Moussavi, Farshid, 2009. *The Function of Form*. Actar and Harvard University Graduate School of Design.

Nervi, Pier Luigi, 1965. *Aesthetics and Technology*. Cambridge, MA: Harvard University Press.

Nervi, Pier Luigi, 1956. *Structures*. New York: F. W. Dodge Corporation.

Nisbet, Michael A., Marceau, Medgar L., & VanGeem, Medgar L., 2003. 'Environmental Life Cycle Inventory of Portland Cement Concrete', PCA R&D Serial No. 2137a, a report on Concrete: Sustainability and Life Cycle, PCA CD033.

Nilson, Arthur H., Darwin, David, & Dolan, Charles W., 2010. *Design of Concrete Structures*. 14th Ed. New York: McGraw-Hill.

Pancoast, Ferendino, Grafton, Skeels, and Burnham, Consulting Architects, 1964. *As Built Construction Drawings for the Marine Stadium*, Miami, FL.

Pazdon, Jennifer Anna, 2009. *Towards the Revitalization of Shell Structures: Design and Analysis of a Prototype for Hyperbolic Paraboloid Shell Construction.* Princeton University, Master's Thesis.

Peurifoy, Robert & Oberlender, Garold, 2011. *Formwork for Concrete Structures*, 4th Ed. McGraw-Hill: New York.

Popular Mechanics, 1930. "Power Plant Cooling Tower Like Big Milk Bottle". February: 201.

Prevost, J., 2012. [Personal Conversation], (January 2012).

Prier, M., 2011. *Responding to Nigeria's Building Challenges: A bamboo dome design for a Nigerian school.* Princeton University undergraduate thesis.

Regalado, Tomás, 2012. [Personal Conversation], (February 2012).

Rodriguez, Eduardo Luis, 2000. *The Havana Guide – Modern Architecture 1925-1965*. New York: Princeton Architectural Press.

Sasser, Michael, 2011. 'Saving the Stadium'. *Miami Sun Post*. X(10). 11 March.

'Sculpture and Structure under Construction: Hilario Candela's, FAIA, Miami Marine Stadium'. *AIA Architect*, 8 October 2010.

Segal, Edward, 2008. *The Thin Concrete Shells of Jack Christiansen*. Princeton University Master's Thesis.

Sebake, TN., 2008. ‘Developing the sustainable building assessment tool for stadia. World Sustainable Building Conference’. Melbourne, Australia, 21-25 September: 8.

Sebake, TN, Gibberd, JT., 2008. PG Report: Sustainable Building Assessment and Support (Unpublished report). CSIR. Pretoria.

Simpson, Gumpertz & Heger, Inc, 1993. “Conditional Apprasial and Structural Review: Miami Marine Stadium Roof Structure”. Miami, Florida, May.

Soroka, I, 1993. *Concrete in Hot Environments*. London: E & FN Spon.

Struble, L. & Godfrey, J., 2004. *How Sustainable is Concrete?*, Proceedings of the International Workshop on Sustainable Development and Concrete Technology, Beijing.

Sustainable Building Assessment Tool (SBAT) for Stadia, 2010. The Council for Scientific and Industrial Research, South Africa.

Tedesko, A., 1980. “How Have Concrete Shell Structures Performed? An Engineer Looks Back at year of Experience with Shells,” *Bulletin No. 73*, IASS.

Thrall, Ashley Parkinson, 2008. *A Comparison of the Work of Gustave Eiffel and Othmar Ammann: The Maria Pia and Bayonne Bridges.* Princeton University Master’s Thesis.

UN Documents. ‘Report on the World Commission on Environment and Development: Our Common Future’. [Online]. Available at: www.un-documents.net/wced-ocf.htm

World Business Council for Sustainable Development (WBCSD), 2005. Website [Online]. Available at: www.wbcsd.org

Yoo, Chai H. and Sung C. Lee, 2011. *Stability of Structures*. Burlington, MA: Elsevier Inc.

Zigo, Tomislav, 2011. “Green Building Studio SD Evaluation”, Presentation to CEE 477, October 2011.

10.Appendices

Appendix A: Efficiency Impact Ratio Calculations

Model Name	Area of Concrete on Roof	Building Footprint	Thickness	Other Miscellaneous Concrete	Total
	(m^2)	(m^2)	(m)	(m^3)	(m^3 per m^2 of floor space)
St. Vincent de Paul	526.58	466.5	0.04		0.045153606
Chapel Lomas de Cuernavaca	805.68	455.1	0.04		0.070813691
Rio Warehouse	5777.53	5490.0	0.04	56.63	0.052410712
Los Manantiales	1607.84	742.3	0.04		0.086642292
Cosmic Rays	184.54	129.0	0.04	4.49	0.092057054
Bacardi Rum Factory	5484.34	5394.0	0.04	78.84	0.061500541
Activity Center	2306.59	2090.3	0.05	61.31	0.084503712

One-way Slab	743.2	743.2	0.2770	12.92	0.294484
Two-way Flat plate	464.5	464.5	0.254	4.08	0.262778
Two-way waffle slab	2191.6	5574.2	Variable	7.93	0.394607

Model Name	Largest Span	Smallest Span	SIR
	(m)	(m)	(lbs. CO_2 / m^2)
St. Vincent de Paul	28.96	18.06	23.62
Chapel Lomas de Cuernavaca	31	18	37.05
Rio Warehouse	14.6	9.65	27.42
Los Manantiales	42.43	5.84	45.33
Cosmic Rays	12	10.25	48.16
Bacardi Rum Factory	31	26	32.18
Activity Center	64.6	32.3	44.21

One-way Slab	6.096	6.096	154.07
Two-way Flat Plate	10.9728	7.62	137.48
Two-way Waffle Slab	16.764	17.76	206.45

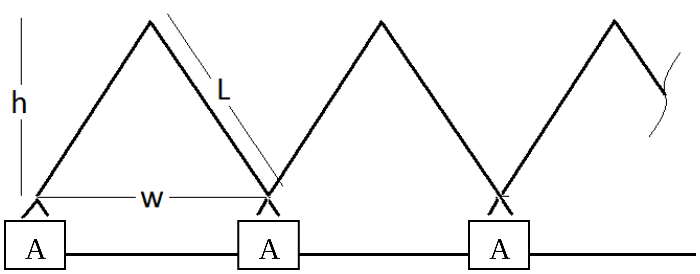
Appendix B: Comparison between FEM parameters and Simpson *et. al.* parameters

Assumptions List	**NCB**		**Simpson *et. al.***		**Units**
Concrete Type	Normal	Lightweight	Normal	Lightweight	
Region Terminology	Backspan	Main Cantilever	Rear hypar	Front hypar	
Young's Modulus	25,100	25,100	24,855	23,250	Mpa
Density of Concrete	2,402	1,601	2,322	1,842	kg/m^3
Self-Weight	Calc. by LUSAS	Calc. by LUSAS	400,000	155,000	lbs.
Self-Weight (Area)	Calc. by LUSAS	Calc. by LUSAS	1,550	2,700	sq. ft.
Live Load	none	none	30	30	psf
Dead Load	Calc. by LUSAS	Calc. by LUSAS	258	57	psf

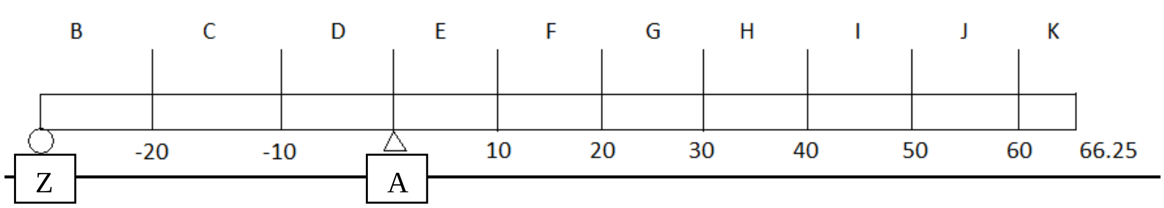
Appendix C: FEM-1 Model Validation Calculations

This FEM-1 model validation is a simple folded plate analysis used to calculate reactions at the main central support. First, the shells are considered as a continuous plate in the *x*-direction. The support reactions from the continuous plate calculations are then loaded onto the cantilever in the *y*-direction, leading to support reaction at the middle umbrella support, here called *A.* Since the cantilever varies in cross section, it is divided into 10 discrete regions. For each region, the weight of the shell and the weight of the groin are added to obtain the support reaction, taking into account the changing groin area and angle of the shell. This support reaction is applied to the cantilever as a lineal live load over that region in the *y*-direction. Further on in the calculations, the lineal live load is broken down into its resultant vector, and standard equations of static equilibrium are used to obtain the support reaction *A*. *A* is assumed to take the weight of one umbrella, which includes one groin, two shells, and the backspan. It is completed in U.S. units, but the final reaction is converted for comparison. For more in-depth hand calculations of the folded-plate action, Adriaenssens *et. al.* (2012) should be consulted.

Continuous plate in *x*-direction:



Cantilevered beam in *y*-direction:



Equations:

$* \, All \; geometry \; from \; Original \; Drawings$

$* \, All \; weights \; given \; as \; lineal \; loads, due \; to \; the \; loading \; of$
$the \; cantilever \; by \; the \; plate \; support \; reactions$

$$\boldsymbol{Weight\ of\ shell} = L * 2 * thickness * density\ of\ concrete$$

$$\boldsymbol{Weight\ of\ groin} = area\ of\ groin * density\ of\ concrete$$

$$\boldsymbol{Slab\ support\ reaction} = weight\ of\ shell + weight\ of\ groin$$

$$\boldsymbol{Centroid} = center\ of\ mass\ of\ discrete\ roof\ strip$$

$$\boldsymbol{Resultant} = Slab\ support\ reaction * size\ of\ roof\ strip$$

$\boldsymbol{Equilibrium}$:

$\sum F_z = 0$ $\qquad\qquad$ $\sum M_A = 0$

$R_A + R_Z = \sum All\ resultants$

$R_Z(-30\ ft) = \sum resultants * ordinates$

R_A = 973 kips = 4168.0 kN

R_A *from FEM* = 848 kips = 3772.0 kN

$* \, difference\ of$ 9.5%

Tables:

	Valley Groin		Peak Groin		Total Groin Area	h	w	L
Region (ft)	Thickness (ft)	Width (ft)	Thickness	Width (ft)	(ft^2 per umbrella)	(ft)	(ft)	(ft)
-30 to -20	Uniform 2 ft thickness					-4.87	40.66	20.91
-20 to -10						2.22	40.66	20.45
-10 to 0						8.47	40.66	22.02
0-10	1.92	2.14	1.83	2.04	7.84	14.25	40.66	24.83
10-20.	1.583	4.72	1.66	4.6	15.11	11.75	40.66	23.48
20-30	1.75	7.74	1.5	7.65	25.02	9.25	40.66	22.34
30-40	1.16	11.25	1.25	11	26.80	6.75	40.66	21.42
40-50	1	14.93	0.92	14.6	28.36	4.25	40.66	20.77
50-60	0.75	18.57	0.75	18.04	27.46	1.75	40.66	20.41
60-66.25	0.25	20.66	0.25	20	10.17	0.00	40.66	20.33

Region	Weight of Shell	Weight of Groins	Slab Support Rxn
	(pounds/ft)	(pounds/ft)	(pounds/ft)
B	12125.48967		12125.48967
C	11860.18317		11860.18317
D	12774.01667		12774.01667
E	1427.542623	901.83	2329.372623
F	1350.174689	1737.39	3087.567089
G	1284.287182	2877.30	4161.587182
H	1231.723819	3082.00	4313.723819
I	1194.245239	3261.63	4455.875239
J	1173.297891	3157.61	4330.910391
K	1168.975	1168.98	2337.95

Region	Distance of Centroid	Size of Range	Resultant	Resultant*Centroid
	(ft)	(ft)	(pounds)	
B	-25	10	133452.9	-3336322.417
C	-15	10	130799.83	-1961997.475
D	-5	10	139938.17	-699690.8333
E	5	10	35491.726	177458.6312
F	15	10	43073.671	646105.0634
G	25	10	53813.872	1345346.796
H	35	10	55335.238	1936733.337
I	45	10	56756.752	2554053.858
J	55	10	55507.104	3052890.715
K	63.125	6.25	22235.938	1403643.555

Rback + Rfront	766.41		
Rback (kips)	-170.61		
Rfront (kips)	937.01	848.00	9.5%
Rback (kN)	-758.90		
Rfront (kN)	4168.04	3772.09	9.5%
FEM-1	**Hand Calcs**	**Model**	**Percent Dif**